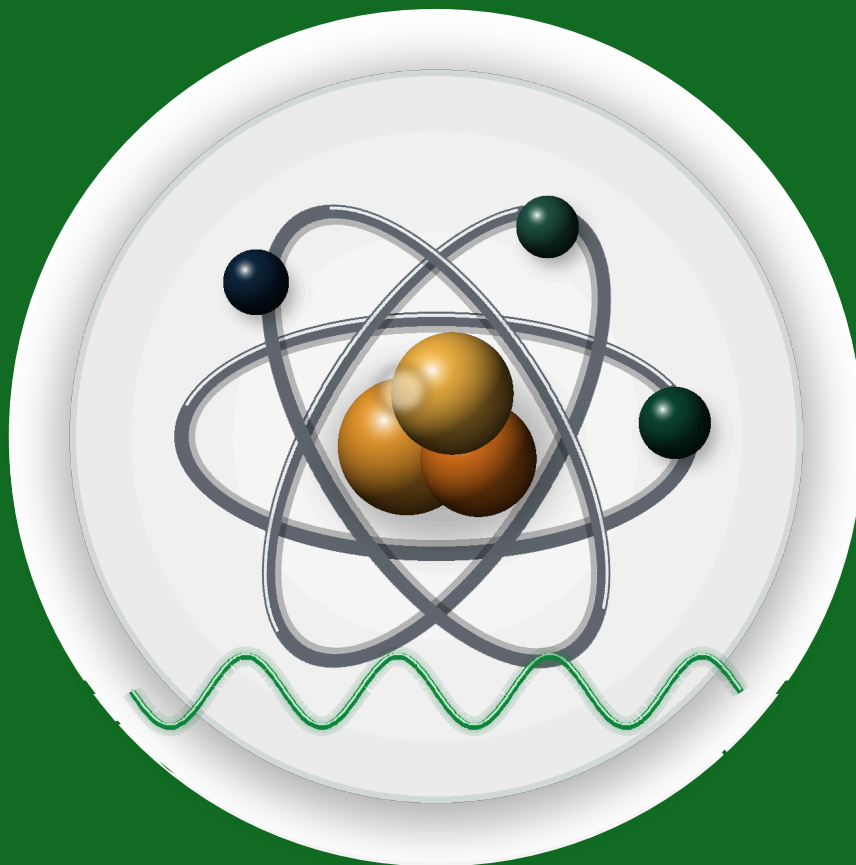


QUANTUM & OPTOELECTRONIC PHYSICS

B E C O M E R E A D Y



HNK



LEARN QOE 5

TEST 1 + TEST 2 COMPLETE PREPARATION



This guide is designed to prepare you for Quantum and Opto-Electronic Physics 5. It combines Test 1 and Test 2 in one calculation-focused revision book, using complete questions, clear figures, formulas, and full mathematical working. The emphasis is on the method needed to earn marks: show each step, keep units, explain important physical reasons, and finish with a clear boxed answer.

OVERVIEW

TEST 1

- Propagation of light and Maxwell wave equations
- Phase and group velocity, wave vectors and phase difference
- Vectorial nature of light and polarization
- Reflection and refraction of electromagnetic waves
- Normal-incidence reflection, Fresnel ideas and optical-fibre applications

TEST 2

- Laser theory, Einstein processes, gain and population inversion
- Black-body radiation, photoelectric effect and Compton scattering
- de Broglie waves, Bohr model and hydrogen transitions
- Schrödinger equation, normalization, expectation values and particle in a box
- Potential steps, tunnelling, harmonic oscillator and hydrogen probability

STUDY METHOD

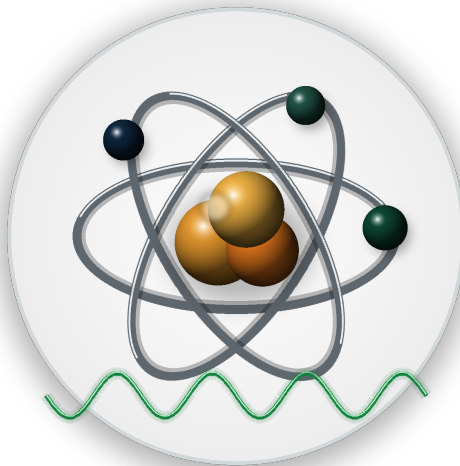
Question → Figure → Given → Find → Formula → Every Step → Boxed Answer

Personal Study Edition

Prepared from the supplied QOE580S tutorials, memos, past tests, chapter notes and study packs. Material is organized for revision and exam preparation, not as a replacement for official course documents.

TEST 1

LIGHT, WAVES & OPTICS



CORE TOPICS

Propagation of Light

Vectorial Nature of Light and Polarization

Reflection and Refraction of EM Waves

QOE580S TEST 1 CALCULATION PREPARATION

TUTORIALS + STEP-BY-STEP PREPARATION

Quantum and Opto-Electronics 5

Strict scope only

Tutorial 1: Propagation of Light

Tutorial 2: Vectorial Nature of Light and Polarization

Tutorial 3: Reflection, Refraction and Optical Fibre

2026 Test 1 paper - 7 October 2026

Calculation-first format

Tutorial / Test Question → Image / Diagram → Resolution → Every Step → Boxed Final Answer

This compact preparation pack includes the full Tutorial 1-3 question bank for Test 1, selected tutorial worked examples, and the supplied 7 October 2026 Test 1. It keeps the calculation-first style: complete question, image or diagram, mathematical resolution, every important step, and a boxed final answer. The separate figure appendix has been removed.

October 2026

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- 2 Compact formula sheet
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 - Tutorial A - Electric-field wave equation
 - Tutorial B - Field-amplitude relation in free space
 - Tutorial C - Fresnel reflection practice
 - Tutorial D - Optical-fibre cutoff practice
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 - Q1.1 Maxwell equations and magnetic-field wave equation
 - Q1.2 Harmonic-wave operator substitutions
 - Q1.3 TE Fresnel reflection coefficient
 - Q2.1 Phase and group velocity
 - Q2.3 Polarization ellipse
 - Q3.1 Plane wave in a dielectric
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- 6 Final rapid memory sheet and exam checklist

Compact edition

The separate Question Figures Appendix is intentionally removed. Figures now appear only where they are needed: directly after the related question and before its calculation.

How to use this pack

The calculation style used throughout is intentionally simple:

1. Read the complete question.
2. Copy the given values and write what must be found.
3. Write the formula before substituting.
4. Convert all units first.
5. Substitute one line at a time.
6. Keep units with numbers.
7. Show calculus, algebra, boundary conditions and reasons.
8. Box the final answer to 3 significant figures unless the question states otherwise.

Exam note

For derivations, marks are normally attached to the intermediate vector identity, boundary condition, algebra and substitution. Do not jump straight to the final formula.

Compact formula sheet

2.1 Maxwell equations and plane waves

$$\begin{aligned}
 \nabla \cdot \mathbf{E} &= 0, & \nabla \cdot \mathbf{B} &= 0, \\
 \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, & \nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \\
 \nabla \times (\nabla \times \mathbf{A}) &= \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}, \\
 \nabla^2 \mathbf{E} &= \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, & \nabla^2 \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}, \\
 \frac{\partial}{\partial t} &\rightarrow -j\omega, & \nabla &\rightarrow j\mathbf{k}, & \mathbf{k} \times \mathbf{E} &= \omega \mathbf{B}, \\
 B &= \frac{E}{v}, & v &= f\lambda = \frac{\omega}{k}, & k &= \frac{2\pi}{\lambda}, & \omega &= 2\pi f.
 \end{aligned}$$

2.2 Dispersion and polarization

$$\begin{aligned}\omega^2 &= c^2 k^2 + \omega_p^2, & v_p &= \frac{\omega}{k}, & v_g &= \frac{d\omega}{dk} \simeq \frac{\Delta\omega}{\Delta k}, \\ E(x, t) &= 2E_0 \cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right) \cos(k_{\text{avg}}x - \omega_{\text{avg}}t), \\ \left(\frac{E_x}{E_1}\right)^2 - 2\left(\frac{E_x}{E_1}\right)\left(\frac{E_y}{E_2}\right) \cos \Delta\phi + \left(\frac{E_y}{E_2}\right)^2 &= \sin^2 \Delta\phi. \\ \Delta\phi &= k\Delta z = \frac{2\pi\Delta z}{\lambda}, & \Delta\phi &= \omega\Delta t = 2\pi f\Delta t.\end{aligned}$$

2.3 Reflection, refraction and optical fibre

$$\begin{aligned}n_1 \sin \theta_i &= n_2 \sin \theta_t, & n &= \frac{n_2}{n_1}, \\ r_s &= \frac{\cos \theta_i - \sqrt{n^2 - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{n^2 - \sin^2 \theta_i}}, \\ r_p &= \frac{-n^2 \cos \theta_i + \sqrt{n^2 - \sin^2 \theta_i}}{n^2 \cos \theta_i + \sqrt{n^2 - \sin^2 \theta_i}}, \\ R_s &= |r_s|^2, & R_p &= |r_p|^2, & R_{\text{avg}} &= \frac{R_s + R_p}{2}, \\ \tan \theta_B &= \frac{n_2}{n_1}, & \theta_c &= \sin^{-1}\left(\frac{n_2}{n_1}\right), \\ \text{NA} &= n_0 \sin \theta_a = \sqrt{n_1^2 - n_2^2} \quad (n_0 = 1 \text{ for air}).\end{aligned}$$

Printed-paper note

The 2026 paper supplies several of these relations. You still earn marks by identifying the correct relation and showing the intermediate substitution clearly.

Full Tutorial 1-3 question bank for Test 1

Use this section as a practice checklist.

These are the tutorial questions that build the methods used in Test 1. Attempt each question first, then use the worked tutorial examples and the Test 1 solutions that follow.

Tutorial 1 - Propagation of Light

1. By taking the curl of both sides of Faraday's Law and applying

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A},$$

derive the free-space electric-field wave equation

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}.$$

2. In a dispersive medium, use the high-frequency approximation

$$\omega^2 = c^2 k^2 + \omega_p^2$$

to: (i) derive the phase velocity $v_p = \omega/k$, (ii) derive the group velocity $v_g = d\omega/dk$, and (iii) prove that $v_p v_g = c^2$.

3. For the electromagnetic spectrum: (a) convert the given frequencies to wavelengths for 60 Hz, AM radio 535 – 1605 kHz, FM radio 88 – 108 MHz, visible light $\sim 10^{14}$ Hz, and X-rays $\sim 10^{18}$ Hz; (b) convert wavelengths 1 km, 1 m, 1 mm, 1 μm , 1 $\text{\AA}$ to temporal frequencies; (c) determine the corresponding spatial frequencies; and (d) express the requested values in electron-volts where required by the tutorial.
4. An electromagnetic wave has spatial frequency $k_0 = 100 K_0$. Determine its wavelength and temporal frequency. Also determine the spatial frequency for laser light at $\lambda = 0.6328 \mu\text{m}$ and for a microwave oven at $f = 2.4 \text{ GHz}$.
5. For

$$\mathbf{E} = \hat{x} E_0 \cos(kz - \omega t),$$

identify the propagation direction and write the corresponding travelling-wave form requested in the tutorial.

6. Two waves are

$$y_1 = A \cos(k_1 x - \omega_1 t), \quad y_2 = A \cos(k_2 x - \omega_2 t),$$

where $k_2 = k_1 + \Delta k$ and $\omega_2 = \omega_1 + \Delta \omega$. Show that their sum can be written as a carrier multiplied by an envelope, then derive v_p and v_g . Finally calculate the velocities for

$$k_1 = 1.00 \times 10^7 \text{ m}^{-1}, \quad \omega_1 = 3.00 \times 10^{15} \text{ rad/s},$$

$$k_2 = 1.02 \times 10^7 \text{ m}^{-1}, \quad \omega_2 = 3.03 \times 10^{15} \text{ rad/s}.$$

Tutorial 2 - Vectorial Nature of Light and Polarization

1. For

$$U(\mathbf{r}, t) = U_0 e^{j(\mathbf{k} \cdot \mathbf{r} - \omega t)},$$

show directly that

$$\frac{\partial U}{\partial t} = -j\omega U, \quad \nabla U = j\mathbf{k}U.$$

Then transform $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$ into algebraic vector form and interpret the result.

2. Using $\nabla \rightarrow j\mathbf{k}$ and $\partial/\partial t \rightarrow -j\omega$, convert Faraday's Law and Ampere-Maxwell Law to vector algebra. Show that $\mathbf{E}$, $\mathbf{B}$, and $\mathbf{k}$ are mutually orthogonal and form a right-handed triad.
3. Starting from

$$\mathbf{k} \times \mathbf{E} = \omega \mathbf{B},$$

show that $B = E/c$ in free space. For $E_0 = 150 \text{ V/m}$, calculate B_0 and H_0 .

4. Derive

$$n = \sqrt{\epsilon_r}$$

for a non-magnetic optical medium. For $\epsilon_r = 2.25$, determine the refractive index and phase velocity.

5. For

$$E_x = E_1 \cos(kz - \omega t + \phi_1), \quad E_y = E_2 \cos(kz - \omega t + \phi_2),$$

eliminate the common time-dependent phase and derive the polarization ellipse

$$\left(\frac{E_x}{E_1}\right)^2 - 2\left(\frac{E_x}{E_1}\right)\left(\frac{E_y}{E_2}\right)\cos(\phi_1 - \phi_2) + \left(\frac{E_y}{E_2}\right)^2 = \sin^2(\phi_1 - \phi_2).$$

State the polarization for the special phase differences requested in the tutorial.

Tutorial 3 - Reflection and Refraction of EM Waves

1. Derive the three-dimensional magnetic-field wave equation

$$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

showing the vector identity and intermediate algebra.

2. A plane travelling wave propagates along z in a dielectric with $\epsilon_r = 2.55$,

$$E_y = E_0 \cos(\omega t - kz), \quad f = 2.4 \text{ GHz}, \quad E_0 = 30 \text{ V/m}.$$

Find the magnetic-field amplitude and direction, the phase velocity, the wavelength, and the phase shift from $z_1 = 0.5 \text{ m}$ to $z_2 = 1.7 \text{ m}$.

3. Light travels from air ($n_1 = 1.00$) to a dielectric ($n_2 = 1.60$). Calculate the Brewster angle, the critical angle for the reverse direction, and the refraction angle when incidence is at Brewster angle. Explain the physical meaning of Brewster incidence.
4. Unpolarized light in air ($n_1 = 1.00$) strikes glass ($n_2 = 1.50$) at 45° . Calculate the refracted angle, the TE and TM Fresnel coefficients, their power reflectances, and the average reflectance.
5. Derive Brewster's angle for TM polarization by setting $r_p = 0$, and obtain the result in terms of $n = n_2/n_1$.
6. An optical fibre has $n_1 = 1.48$, $n_2 = 1.44$, and air outside. Find the critical angle, the maximum acceptance angle, and the numerical aperture.
7. A step-index fibre has $n_1 = 1.50$, $n_2 = 1.47$, $n_0 = 1.000$, and core diameter $d = 5.0 \mu\text{m}$. Find the critical angle, numerical aperture, and the cutoff wavelength for single-mode operation using $V \leq 2.405$.

Tutorial-to-test link

Tutorial 1 builds the Maxwell-wave and dispersion methods. Tutorial 2 builds the harmonic-wave, field-relation and polarization methods. Tutorial 3 builds the dielectric, Fresnel, Brewster and optical-fibre methods used directly in the 2026 Test 1.

Test 1 preparation map - Tutorials 1 to 3

Tutorial	What to practise for Test 1
Tutorial 1	Maxwell wave-equation derivation; plasma dispersion; phase and group velocity; EM-spectrum conversions; two-wave packet superposition.
Tutorial 2	Plane-wave operator substitutions; Maxwell curl equations in algebraic form; $\mathbf{E}, \mathbf{B}, \mathbf{k}$ orthogonality; field-amplitude relation; $n = \sqrt{\epsilon_r}$; polarization ellipse.
Tutorial 3	Magnetic-field wave equation; dielectric-wave calculations; Snell law; Fresnel coefficients and reflectance; Brewster angle; critical angle; acceptance angle; numerical aperture; fibre cutoff condition.

Best preparation method

First attempt the tutorial question without looking at the solution. Then compare your sequence of formulas and algebra with the worked method. Finally, attempt the matching Test 1 question.

Tutorial A - Electric-field wave equation

Source: Tutorial 1, Problem 1 **Complete question.** By taking the curl of Faraday's law and using $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$, derive the free-space wave equation for the electric field $\mathbf{E}$.

Diagram / Figure



Resolution / Calculation

Step 1. Start with $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$.

Step 2. Take the curl: $\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B})$.

Step 3. Use the vector identity and $\nabla \cdot \mathbf{E} = 0$:

$$-\nabla^2 \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}).$$

Step 4. Use $\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$.

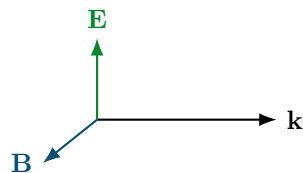
Final answer

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Tutorial B - Field-amplitude relation in free space

Source: *Tutorial 2, Problem 3* **Complete question.** Starting from $\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$, show that $B = E/c$. Then, for $E_0 = 150 \text{ V/m}$, calculate B_0 and H_0 .

Diagram / Figure



Resolution / Calculation

Step 1. For mutually perpendicular $\mathbf{k}$ and $\mathbf{E}$, $|\mathbf{k} \times \mathbf{E}| = kE$, so $kE = \omega B$.

Step 2. Hence

$$B = \frac{k}{\omega} E = \frac{E}{v}.$$

In free space $v = c$, therefore $B = E/c$.

Step 3. For $E_0 = 150 \text{ V/m}$:

$$B_0 = \frac{150}{3.00 \times 10^8} = 5.00 \times 10^{-7} \text{ T}.$$

Step 4. Using $H_0 = B_0/\mu_0$:

$$H_0 = \frac{5.00 \times 10^{-7}}{4\pi \times 10^{-7}} = 0.398 \text{ A/m}.$$

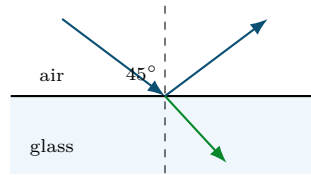
Final answer

$$B_0 = 5.00 \times 10^{-7} \text{ T}, \quad H_0 = 0.398 \text{ A/m}$$

Tutorial C - Fresnel reflection practice

Source: Tutorial 3, Problem 4 **Complete question.** Unpolarized light in air ($n_1 = 1.00$) strikes glass ($n_2 = 1.50$) at $\theta_i = 45^\circ$. Find the refraction angle, r_s , r_p , R_s , R_p , and R_{avg} .

Diagram / Figure



Resolution / Calculation

Step 1. Snell law:

$$\theta_t = \sin^{-1} \left(\frac{1.00}{1.50} \sin 45^\circ \right) = 28.1^\circ.$$

Step 2. With $n = 1.50$,

$$\sqrt{n^2 - \sin^2 45^\circ} = 1.323.$$

Step 3. Therefore

$$r_s = \frac{0.7071 - 1.323}{0.7071 + 1.323} = -0.303,$$

$$r_p = \frac{-2.25(0.7071) + 1.323}{2.25(0.7071) + 1.323} = -0.0920.$$

Step 4.

$$R_s = |r_s|^2 = 9.21\%, \quad R_p = |r_p|^2 = 0.846\%,$$

$$R_{\text{avg}} = \frac{R_s + R_p}{2} = 5.03\%.$$

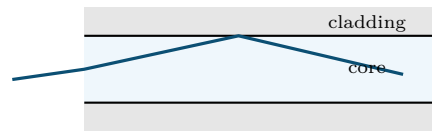
Final answer

$$\theta_t = 28.1^\circ, \quad r_s = -0.303, \quad r_p = -0.0920, \quad R_{\text{avg}} = 5.03\%$$

Tutorial D - Optical-fibre cutoff practice

Source: Tutorial 3, Problem 7 **Complete question.** A step-index fibre has $n_1 = 1.50$, $n_2 = 1.47$, air outside, and core diameter $d = 5.0 \mu\text{m}$. Find the critical angle, numerical aperture, and cutoff wavelength for single-mode operation using $V = \pi d \text{NA} / \lambda$ with cutoff $V = 2.405$.

Diagram / Figure



Resolution / Calculation

Step 1. Critical angle:

$$\theta_c = \sin^{-1}\left(\frac{1.47}{1.50}\right) = 78.5^\circ.$$

Step 2. Numerical aperture:

$$\text{NA} = \sqrt{1.50^2 - 1.47^2} = 0.298.$$

Step 3. At cutoff, $V = 2.405$:

$$\lambda_c = \frac{\pi d \text{NA}}{2.405} = \frac{\pi(5.0 \times 10^{-6})(0.298)}{2.405} = 1.95 \times 10^{-6} \text{ m}.$$

Final answer

$$\theta_c = 78.5^\circ, \quad \text{NA} = 0.298, \quad \lambda_c = 1.95 \mu\text{m} = 1950 \text{ nm}$$

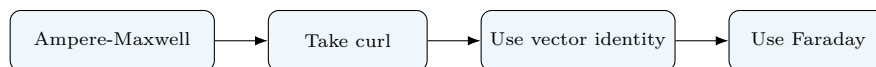
2026 Test 1 - worked questions

Question 1 - Maxwell equations and magnetic-field wave equation

Source: QOE Test 1, 7 October 2026, Q1.1

Complete question. (a) Write the four fundamental Maxwell equations governing electromagnetic fields in free space. (b) Derive the three-dimensional wave equation for the magnetic field vector $\mathbf{B}$ by taking the curl of Ampere-Maxwell law and using the appropriate vector identity.

Diagram / Figure



Resolution / Calculation

(a) Maxwell equations in free space

$$\nabla \cdot \mathbf{E} = 0, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

(b) **Derivation Step 1.** Start with Ampere-Maxwell law: $\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \partial \mathbf{E} / \partial t$.

Step 2. Take the curl: $\nabla \times (\nabla \times \mathbf{B}) = \mu_0 \epsilon_0 \partial (\nabla \times \mathbf{E}) / \partial t$.

Step 3. Use $\nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B}$. Since $\nabla \cdot \mathbf{B} = 0$, the left side becomes $-\nabla^2 \mathbf{B}$.

Step 4. Use Faraday law: $\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$. Then $-\nabla^2 \mathbf{B} = -\mu_0 \epsilon_0 \partial^2 \mathbf{B} / \partial t^2$.

Final answer

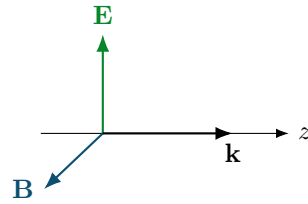
$$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Question 2 - Plane harmonic wave operator substitutions

Source: QOE Test 1, 7 October 2026, Q1.2

Complete question. For a plane monochromatic wave $\mathbf{U}(\mathbf{r}, t) = \mathbf{U}_0 e^{j(\mathbf{k} \cdot \mathbf{r} - \omega t)}$, show by differentiation that $\partial/\partial t \rightarrow -j\omega$ and $\nabla \rightarrow j\mathbf{k}$. Then convert a Maxwell curl relation into vector-algebra form.

Diagram / Figure



Resolution / Calculation

Step 1. $\partial \mathbf{U} / \partial t = (-j\omega) \mathbf{U}$, hence $\partial / \partial t \rightarrow -j\omega$.

Step 2. Because $\mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z$, $\partial \mathbf{U} / \partial x = jk_x \mathbf{U}$, similarly for y, z . Therefore $\nabla \mathbf{U} = j\mathbf{k} \mathbf{U}$ and $\nabla \rightarrow j\mathbf{k}$.

Step 3. Apply these to Faraday law $\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$: $j\mathbf{k} \times \mathbf{E} = j\omega \mathbf{B}$.

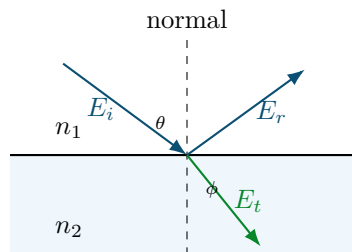
Final answer

$$\frac{\partial}{\partial t} \rightarrow -j\omega, \quad \nabla \rightarrow j\mathbf{k}, \quad \mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$$

Question 3 - TE Fresnel reflection coefficient

Source: QOE Test 1, 7 October 2026, Q1.3 **Complete question.** Show that the TE Fresnel reflection coefficient can be simplified to $r_s = [\cos \theta - \sqrt{n^2 - \sin^2 \theta}] / [\cos \theta + \sqrt{n^2 - \sin^2 \theta}]$, where $n = n_2 / n_1$.

Diagram / Figure



Resolution / Calculation

Step 1. Tangential E continuity gives $E_i + E_r = E_t$.

Step 2. Tangential H continuity gives $n_1 \cos \theta (E_i - E_r) = n_2 \cos \phi E_t$.

Step 3. Substitute $E_t = E_i + E_r$ and solve for $r_s = E_r / E_i$: $r_s = (n_1 \cos \theta - n_2 \cos \phi) / (n_1 \cos \theta + n_2 \cos \phi)$.

Step 4. From Snell's law, $\sin \phi = \sin \theta / n$, so $n_2 \cos \phi = n_1 \sqrt{n^2 - \sin^2 \theta}$.

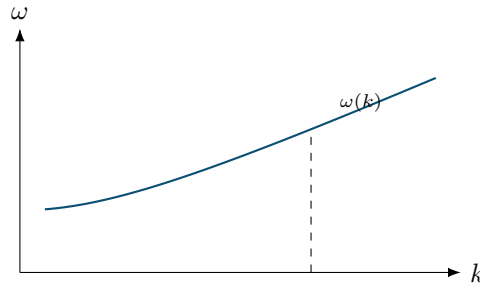
Final answer

$$r_s = \frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}}$$

Question 4 - Phase velocity, group velocity and wave packet

Source: QOE Test 1, 7 October 2026, Q2.1 **Complete question.** In a plasma, $\omega^2 = c^2 k^2 + \omega_p^2$. Derive v_p and v_g , prove $v_p v_g = c^2$, and calculate the packet group velocity for $k_1 = 2.00 \times 10^7 \text{ m}^{-1}$, $\omega_1 = 6.00 \times 10^{15} \text{ rad/s}$, $k_2 = 2.04 \times 10^7 \text{ m}^{-1}$, $\omega_2 = 6.06 \times 10^{15} \text{ rad/s}$.

Diagram / Figure



Resolution / Calculation

Step 1. $\omega = \sqrt{c^2 k^2 + \omega_p^2}$, hence $v_p = \omega/k = \sqrt{c^2 + \omega_p^2/k^2}$.

Step 2. Differentiate $\omega^2 = c^2 k^2 + \omega_p^2$: $2\omega d\omega/dk = 2c^2 k$, so $v_g = d\omega/dk = c^2 k/\omega$.

Step 3. $v_p v_g = (\omega/k)(c^2 k/\omega) = c^2$.

Step 4. For the two signals, $v_g \simeq \frac{\Delta\omega}{\Delta k} = \frac{(6.06 - 6.00) \times 10^{15}}{(2.04 - 2.00) \times 10^7} = 1.50 \times 10^8 \text{ m/s}$.

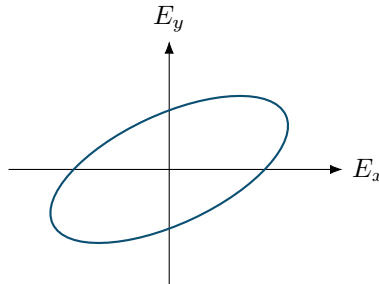
Final answer

$$v_p = \frac{\sqrt{c^2 k^2 + \omega_p^2}}{k}, \quad v_g = \frac{c^2 k}{\omega}, \quad v_p v_g = c^2, \quad v_g \approx 1.50 \times 10^8 \text{ m/s}$$

Question 5 - Polarization ellipse

Source: QOE Test 1, 7 October 2026, Q2.3 **Complete question.** For $E_x = E_1 \cos(kz - \omega t + \phi_1)$ and $E_y = E_2 \cos(kz - \omega t + \phi_2)$, eliminate the common phase and show the general polarization ellipse. State the polarization for $\Delta\phi = 0$, and for $\Delta\phi = \pi/2$ with $E_1 = E_2$.

Diagram / Figure



Resolution / Calculation

Step 1. Let $\psi = kz - \omega t$, $X = E_x/E_1$, $Y = E_y/E_2$. Then $X = \cos(\psi + \phi_1)$ and $Y = \cos(\psi + \phi_2)$.

Step 2. Expand both cosine terms: $X = \cos\psi \cos\phi_1 - \sin\psi \sin\phi_1$ and $Y = \cos\psi \cos\phi_2 - \sin\psi \sin\phi_2$.

Step 3. Eliminate $\cos\psi$ and $\sin\psi$, square the resulting pair and add. Using $\cos^2\psi + \sin^2\psi = 1$ gives $X^2 - 2XY \cos(\phi_1 - \phi_2) + Y^2 = \sin^2(\phi_1 - \phi_2)$.

Step 4. With $\Delta\phi = \phi_1 - \phi_2$, substitute back X, Y .

Final answer

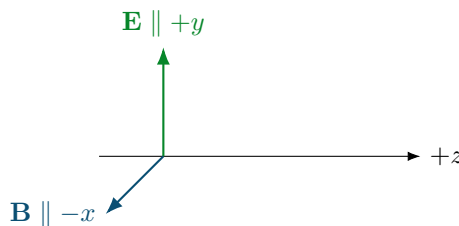
$$\left(\frac{E_x}{E_1}\right)^2 - 2\left(\frac{E_x}{E_1}\right)\left(\frac{E_y}{E_2}\right)\cos\Delta\phi + \left(\frac{E_y}{E_2}\right)^2 = \sin^2\Delta\phi$$

$\Delta\phi = 0$: linear polarization. $\Delta\phi = \pi/2$, $E_1 = E_2$: circular polarization.

Question 6 - Plane wave in a dielectric

Source: QOE Test 1, 7 October 2026, Q3.1 **Complete question.** A plane wave travels in $+z$ inside a non-magnetic dielectric with $\epsilon_r = 3.24$, $E_y = 45 \cos(\omega t - kz)$ V/m and $f = 5.0$ GHz. Find the refractive index, phase velocity, wavelength, magnetic-field amplitude and phase shift between $z_1 = 0.20$ m and $z_2 = 0.80$ m.

Diagram / Figure



Resolution / Calculation

Step 1. For a non-magnetic dielectric, $n = \sqrt{\epsilon_r} = \sqrt{3.24} = 1.80$.

Step 2. $v = c/n = (3.00 \times 10^8)/1.80 = 1.67 \times 10^8$ m/s.

Step 3. $\lambda = v/f = (1.67 \times 10^8)/(5.00 \times 10^9) = 3.33 \times 10^{-2}$ m.

Step 4. $B_0 = E_0/v = 45/(1.67 \times 10^8) = 2.70 \times 10^{-7}$ T. Since $\hat{z} \times \hat{y} = -\hat{x}$, the magnetic field points in $-x$.

Step 5. $\Delta z = 0.60$ m, $k = 2\pi/\lambda = 188.5$ rad/m, so $\Delta\phi = k\Delta z = 113$ rad $= 36\pi$ rad.

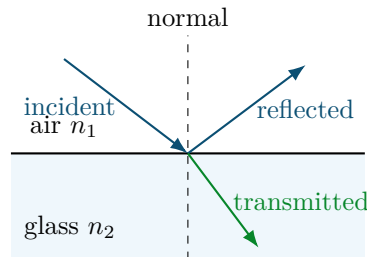
Final answer

$$n = 1.80, v = 1.67 \times 10^8 \text{ m/s}, \lambda = 3.33 \text{ cm}, B_0 = 2.70 \times 10^{-7} \text{ T}, \Delta\phi = 113 \text{ rad}$$

Question 7 - Air-glass Fresnel reflection and Brewster angle

Source: QOE Test 1, 7 October 2026, Q3.2 **Complete question.** Unpolarized light in air ($n_1 = 1.00$) strikes glass ($n_2 = 1.52$) at $\theta_i = 60^\circ$. Find θ_t , the TE and TM reflection coefficients, the reflectance of each polarization and the average reflectance. Then derive and calculate Brewster's angle.

Diagram / Figure



Resolution / Calculation

Step 1. Snell law: $\sin \theta_t = (1/1.52) \sin 60^\circ = 0.56975$, so $\theta_t = 34.7^\circ$.

Step 2. Let $n = 1.52$. Then $\sqrt{n^2 - \sin^2 60^\circ} = 1.249$.

Step 3. $r_s = (0.500 - 1.249)/(0.500 + 1.249) = -0.428$.

Step 4. $r_p = [-1.52^2(0.500) + 1.249]/[1.52^2(0.500) + 1.249] = 0.0391$.

Step 5. $R_s = 0.183 = 18.3\%$, $R_p = 0.00153 = 0.153\%$, and $R_{\text{avg}} = (R_s + R_p)/2 = 0.0925 = 9.25\%$.

Step 6. At Brewster incidence set $r_p = 0$. This gives $\tan \theta_B = n_2/n_1$, hence $\theta_B = \tan^{-1}(1.52) = 56.7^\circ$.

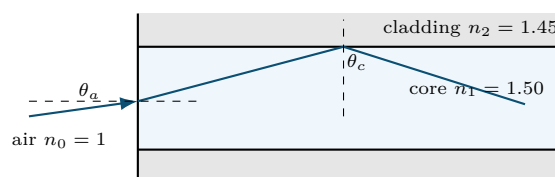
Final answer

$$\theta_t = 34.7^\circ, r_s = -0.428, r_p = 0.0391, R_s = 18.3\%, R_p = 0.153\%, R_{\text{avg}} = 9.25\%, \theta_B = 56.7^\circ$$

Question 8 - Step-index optical fibre

Source: QOE Test 1, 7 October 2026, Q3.3 **Complete question.** A step-index fibre has core index $n_1 = 1.50$, cladding index $n_2 = 1.45$ and air outside ($n_0 = 1.00$). Find the critical angle at the core-cladding boundary, derive and calculate the maximum acceptance angle in air, and calculate the numerical aperture.

Diagram / Figure



Resolution / Calculation

Step 1. At the limiting condition for total internal reflection, $\sin \theta_c = n_2/n_1 = 1.45/1.50 = 0.96667$, so $\theta_c = 75.2^\circ$.

Step 2. At the input face, $n_0 \sin \theta_a = n_1 \sin \theta_r$. At limiting TIR, $\theta_r = 90^\circ - \theta_c$, so $\sin \theta_r = \cos \theta_c$.

Step 3. Therefore $n_0 \sin \theta_a = n_1 \cos \theta_c = \sqrt{n_1^2 - n_2^2}$.

Step 4. $\text{NA} = \sqrt{1.50^2 - 1.45^2} = \sqrt{0.1475} = 0.384$.

Step 5. With $n_0 = 1$, $\sin \theta_a = 0.384$, hence $\theta_a = 22.6^\circ$.

Final answer

$$\theta_c = 75.2^\circ, \quad \theta_a = 22.6^\circ, \quad \text{NA} = 0.384$$

Additional worked tutorial questions

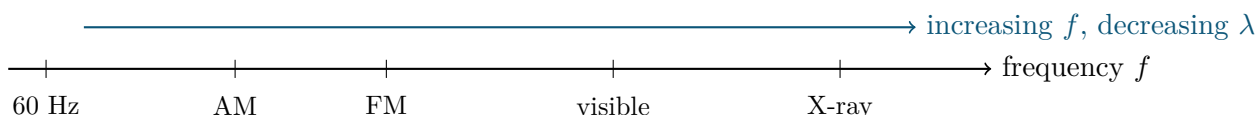
These additions complete the unique Tutorial 1-3 calculation methods that were listed in the full question bank but were not yet worked in full. Questions whose method is already worked in Tutorials A-D or in the 2026 Test 1 section are not duplicated.

Tutorial E - Electromagnetic spectrum calculations

Source:

Tutorial 1, Problem 3 **Complete question.** Convert the specified electromagnetic frequencies to wavelengths, convert the specified wavelengths to temporal frequencies, determine the corresponding spatial frequencies, and express the photon energies in electron-volts where requested.

Diagram / Figure



Resolution / Calculation

Step 1. Use

$$\lambda = \frac{c}{f}, \quad f = \frac{c}{\lambda}, \quad k_0 = \frac{2\pi}{\lambda}, \quad E = \frac{hc}{\lambda}.$$

Take $c = 3.00 \times 10^8$ m/s and $hc \approx 1240$ eV nm.

Step 2. Frequency $\rightarrow$ wavelength:

$$\lambda_{60\text{ Hz}} = \frac{3.00 \times 10^8}{60} = 5.00 \times 10^6 \text{ m}.$$

AM band:

$$\lambda(535 \text{ kHz}) = 561 \text{ m}, \quad \lambda(1605 \text{ kHz}) = 187 \text{ m}.$$

FM band:

$$\lambda(88 \text{ MHz}) = 3.41 \text{ m}, \quad \lambda(108 \text{ MHz}) = 2.78 \text{ m}.$$

Using the tutorial values,

$$\lambda(10^{14} \text{ Hz}) = 3.00 \times 10^{-6} \text{ m}, \quad \lambda(10^{18} \text{ Hz}) = 3.00 \times 10^{-10} \text{ m}.$$

Step 3. Wavelength $\rightarrow$ frequency, spatial frequency and photon energy:

λ	$f = c/\lambda$	$k_0 = 2\pi/\lambda$	$E = hc/\lambda$
1 km	$3.00 \times 10^5 \text{ Hz}$	$6.28 \times 10^{-3} \text{ m}^{-1}$	$1.24 \times 10^{-9} \text{ eV}$
1 m	$3.00 \times 10^8 \text{ Hz}$	6.28 m^{-1}	$1.24 \times 10^{-6} \text{ eV}$
1 mm	$3.00 \times 10^{11} \text{ Hz}$	$6.28 \times 10^3 \text{ m}^{-1}$	$1.24 \times 10^{-3} \text{ eV}$
1 μm	$3.00 \times 10^{14} \text{ Hz}$	$6.28 \times 10^6 \text{ m}^{-1}$	1.24 eV
1 $\text{\AA}$	$3.00 \times 10^{18} \text{ Hz}$	$6.28 \times 10^{10} \text{ m}^{-1}$	$1.24 \times 10^4 \text{ eV}$

Final answer

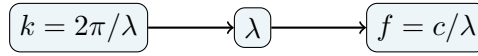
Use the four relations above after converting every prefix to SI. The numerical results are summarized in the table.

Tutorial F - Specific wave conversions

Source:

Tutorial 1, Problem 4 Complete question. An electromagnetic wave has spatial frequency $k_0 = 100$ (interpreted by the supplied memo as 100 rad/m). Determine its wavelength and temporal frequency. Then determine the spatial frequency of laser light at $\lambda = 0.6328 \mu\text{m}$ and of a microwave at $f = 2.4 \text{ GHz}$.

Diagram / Figure



Resolution / Calculation

Step 1. For $k_0 = 100 \text{ rad/m}$,

$$\lambda = \frac{2\pi}{k_0} = \frac{2\pi}{100} = 6.28 \times 10^{-2} \text{ m}.$$

Then

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8}{6.28 \times 10^{-2}} = 4.77 \times 10^9 \text{ Hz} = 4.77 \text{ GHz}.$$

Step 2. Laser light, $\lambda = 0.6328 \mu\text{m} = 6.328 \times 10^{-7} \text{ m}$:

$$k_0 = \frac{2\pi}{\lambda} = \frac{2\pi}{6.328 \times 10^{-7}} = 9.93 \times 10^6 \text{ rad/m}.$$

Step 3. Microwave, $f = 2.4 \text{ GHz}$:

$$\lambda = \frac{3.00 \times 10^8}{2.4 \times 10^9} = 0.125 \text{ m},$$

$$k_0 = \frac{2\pi}{0.125} = 50.3 \text{ rad/m}.$$

Final answer

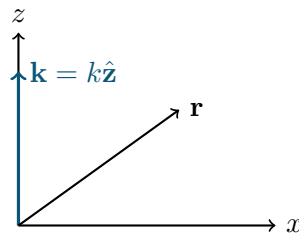
$\lambda = 6.28 \times 10^{-2} \text{ m}$, $f = 4.77 \text{ GHz}$; laser $k_0 = 9.93 \times 10^6 \text{ rad/m}$; microwave $k_0 = 50.3 \text{ rad/m}$.

Tutorial G - Wave-vector notation

Source:

Tutorial 1, Problem 5 Complete question. For a wave written as $\mathbf{E} = \hat{\mathbf{x}}E_0 \cos(kz - \omega t)$ and propagating along $+z$, rewrite the phase using the wave vector and position vector.

Diagram / Figure



Resolution / Calculation

Step 1. Propagation along $+z$ gives

$$\mathbf{k} = k\hat{\mathbf{z}}.$$

Step 2. The position vector is

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}.$$

Step 3. Therefore

$$\mathbf{k} \cdot \mathbf{r} = (k\hat{\mathbf{z}}) \cdot (x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}) = kz.$$

Step 4. Hence

$$\mathbf{E}(\mathbf{r}, t) = \hat{\mathbf{x}}E_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t).$$

Final answer

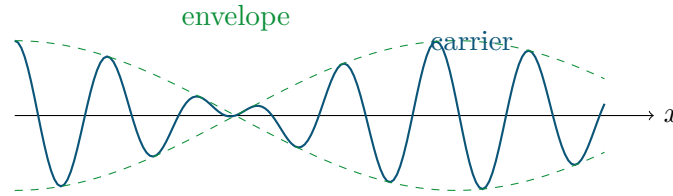
$$\mathbf{E}(\mathbf{r}, t) = \hat{\mathbf{x}}E_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t), \quad \mathbf{k} = k\hat{\mathbf{z}}$$

Tutorial H - Superposition, envelope and wave velocities

Source:

Tutorial 1, Problem 6 Complete question. Two waves are $y_1 = A \cos(k_1 x - \omega_1 t)$ and $y_2 = A \cos(k_2 x - \omega_2 t)$, with $k_2 = k_1 + \Delta k$ and $\omega_2 = \omega_1 + \Delta \omega$. Show that their sum is a carrier multiplied by an envelope, then derive the phase and group velocities. Finally calculate them for $k_1 = 1.00 \times 10^7 \text{ m}^{-1}$, $\omega_1 = 3.00 \times 10^{15} \text{ rad/s}$, $k_2 = 1.02 \times 10^7 \text{ m}^{-1}$, $\omega_2 = 3.03 \times 10^{15} \text{ rad/s}$.

Diagram / Figure



Resolution / Calculation

Step 1. Use

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha - \beta}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right).$$

Let $\alpha = k_1 x - \omega_1 t$ and $\beta = k_2 x - \omega_2 t$. Then

$$\frac{\alpha - \beta}{2} = -\frac{\Delta k}{2} x + \frac{\Delta \omega}{2} t.$$

Since $\cos(-u) = \cos u$,

$$\cos \left(\frac{\alpha - \beta}{2} \right) = \cos \left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t \right).$$

Also define

$$k_{\text{avg}} = \frac{k_1 + k_2}{2}, \quad \omega_{\text{avg}} = \frac{\omega_1 + \omega_2}{2}.$$

Therefore

$$y = 2A \cos \left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t \right) \cos(k_{\text{avg}} x - \omega_{\text{avg}} t).$$

Step 2. Carrier phase velocity:

$$v_p = \frac{\omega_{\text{avg}}}{k_{\text{avg}}}.$$

Envelope group velocity:

$$v_g \approx \frac{\Delta \omega}{\Delta k}.$$

Step 3. Numerical values:

$$k_{\text{avg}} = 1.01 \times 10^7 \text{ m}^{-1}, \quad \omega_{\text{avg}} = 3.015 \times 10^{15} \text{ rad/s}.$$

$$v_p = \frac{3.015 \times 10^{15}}{1.01 \times 10^7} = 2.99 \times 10^8 \text{ m/s}.$$

$$v_g = \frac{(3.03 - 3.00) \times 10^{15}}{(1.02 - 1.00) \times 10^7} = 1.50 \times 10^8 \text{ m/s}.$$

Final answer

$$y = 2A \cos \left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t \right) \cos(k_{\text{avg}} x - \omega_{\text{avg}} t)$$

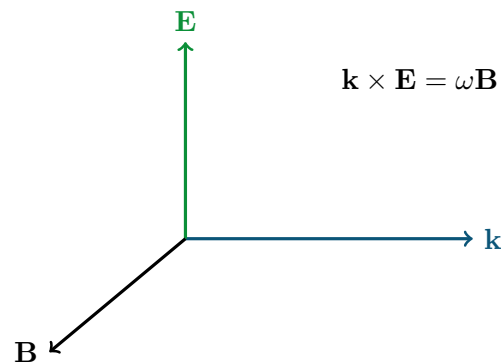
$$v_p = 2.99 \times 10^8 \text{ m/s}, \quad v_g = 1.50 \times 10^8 \text{ m/s}$$

Tutorial I - Maxwell curl equations and wave orthogonality

Source:

Tutorial 2, Problem 2 **Complete question.** Using $\nabla \rightarrow j\mathbf{k}$ and $\partial/\partial t \rightarrow -j\omega$, convert Faraday's law and Ampere-Maxwell law to vector algebra and show that $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{k}$ form a mutually orthogonal right-handed triad.

Diagram / Figure



Resolution / Calculation

Step 1. Faraday law:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Substitute the harmonic operators:

$$j\mathbf{k} \times \mathbf{E} = -(-j\omega\mathbf{B}) = j\omega\mathbf{B}.$$

Cancel j :

$$\boxed{\mathbf{k} \times \mathbf{E} = \omega\mathbf{B}}.$$

Step 2. Ampere-Maxwell law:

$$\nabla \times \mathbf{B} = \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Thus

$$j\mathbf{k} \times \mathbf{B} = -j\omega\mu_0\epsilon_0\mathbf{E},$$

so

$$\boxed{\mathbf{k} \times \mathbf{B} = -\omega\mu_0\epsilon_0\mathbf{E}}.$$

Step 3. From $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$:

$$\mathbf{k} \cdot \mathbf{E} = 0, \quad \mathbf{k} \cdot \mathbf{B} = 0.$$

Therefore both fields are transverse to $\mathbf{k}$. **Step 4.** Because $\mathbf{k} \times \mathbf{E}$ is parallel to $\mathbf{B}$, the three vectors are mutually perpendicular. Equivalently $\mathbf{E} \times \mathbf{B}$ points along the propagation direction.

Final answer

$$\boxed{\mathbf{k} \cdot \mathbf{E} = 0, \quad \mathbf{k} \cdot \mathbf{B} = 0, \quad \mathbf{k} \times \mathbf{E} = \omega\mathbf{B}}$$

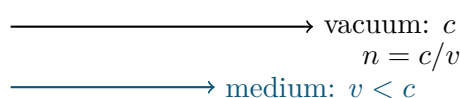
$\mathbf{E}$, $\mathbf{B}$ and $\mathbf{k}$ form a right-handed orthogonal triad.

Tutorial J - Refractive index and relative permittivity

Source:

Tutorial 2, Problem 4 **Complete question.** Derive the relationship between refractive index n and relative permittivity ϵ_r for a non-magnetic optical medium. Then find n and the phase velocity for $\epsilon_r = 2.25$.

Diagram / Figure



Resolution / Calculation

Step 1. Start from

$$n = \frac{c}{v}.$$

For a material,

$$v = \frac{1}{\sqrt{\mu\epsilon}}, \quad c = \frac{1}{\sqrt{\mu_0\epsilon_0}}.$$

Hence

$$n = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} = \sqrt{\mu_r\epsilon_r}.$$

Step 2. For a non-magnetic optical material, $\mu_r \approx 1$:

$$\boxed{n = \sqrt{\epsilon_r}}.$$

Step 3. With $\epsilon_r = 2.25$,

$$n = \sqrt{2.25} = 1.50.$$

$$v = \frac{c}{n} = \frac{3.00 \times 10^8}{1.50} = 2.00 \times 10^8 \text{ m/s}.$$

Final answer

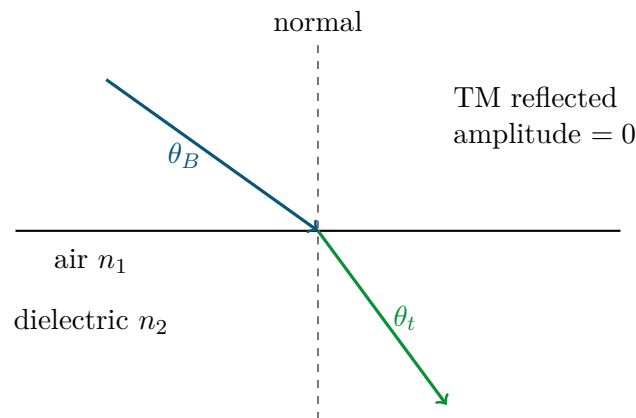
$$\boxed{n = 1.50, \quad v = 2.00 \times 10^8 \text{ m/s}}$$

Tutorial K - Brewster angle, critical angle and refraction

Source:

Tutorial 3, Problem 3 **Complete question.** Light travels from air ($n_1 = 1.00$) to a dielectric ($n_2 = 1.60$). Calculate the Brewster angle, the critical angle for the reverse direction, and the refraction angle when the incidence angle is the Brewster angle. Explain the physical meaning of Brewster incidence.

Diagram / Figure



Resolution / Calculation

Step 1. Brewster angle:

$$\tan \theta_B = \frac{n_2}{n_1} = 1.60.$$

$$\theta_B = \tan^{-1}(1.60) = 58.0^\circ.$$

Step 2. Critical angle for dielectric $\rightarrow$ air:

$$\sin \theta_c = \frac{n_{\text{air}}}{n_{\text{dielectric}}} = \frac{1.00}{1.60} = 0.625.$$

$$\theta_c = \sin^{-1}(0.625) = 38.7^\circ.$$

Step 3. Refraction angle at Brewster incidence. Use Snell law:

$$n_1 \sin \theta_B = n_2 \sin \theta_t.$$

$$\sin \theta_t = \frac{1.00}{1.60} \sin 58.0^\circ = 0.530.$$

$$\theta_t = 32.0^\circ.$$

As expected at Brewster incidence,

$$\theta_B + \theta_t \approx 90^\circ.$$

Step 4. Physical meaning: at θ_B , the TM (p-polarized) reflection coefficient is zero. An initially unpolarized reflected beam is therefore ideally TE (s) polarized.

Final answer

$$\theta_B = 58.0^\circ, \quad \theta_c = 38.7^\circ, \quad \theta_t = 32.0^\circ$$

Tutorial completeness note. Tutorials A-D plus Tutorials E-K now cover the unique worked methods from the Tutorial 1-3 bank. Methods repeated directly in the 2026 Test 1 questions are intentionally not duplicated a second time.

Final rapid memory sheet

Waves / polarization

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \partial^2 \mathbf{E} / \partial t^2$$

$$\partial / \partial t \rightarrow -j\omega, \nabla \rightarrow j\mathbf{k}$$

$$\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$$

$$v_p = \omega / k, v_g = d\omega / dk$$

$$\Delta\phi = k\Delta z = 2\pi\Delta z / \lambda$$

Polarization ellipse equation from E_x, E_y

Interfaces / fibre

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$R_s = |r_s|^2, R_p = |r_p|^2$$

$$R_{\text{avg}} = (R_s + R_p) / 2$$

$$\tan \theta_B = n_2 / n_1$$

$$\sin \theta_c = n_2 / n_1$$

$$\text{NA} = \sqrt{n_1^2 - n_2^2} \text{ in air}$$

Source coverage index - Test 1 only

This index shows where every question from the photographed 7 October 2026 paper is covered.

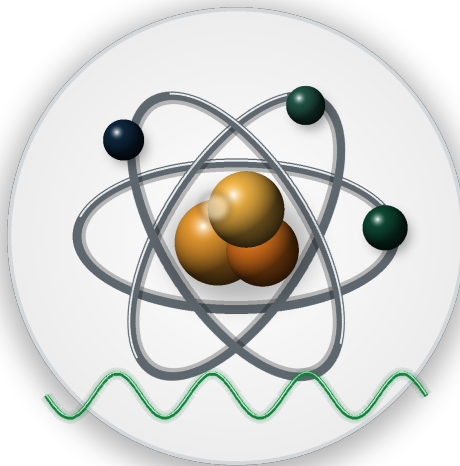
Source item	Topic	Covered here
QOE Test 1 2026 Q1.1	Maxwell equations; magnetic-field wave equation	Question 1
QOE Test 1 2026 Q1.2	Harmonic-wave operator substitutions	Question 2
QOE Test 1 2026 Q1.3	TE Fresnel derivation	Question 3
QOE Test 1 2026 Q2.1	Plasma phase/group velocity and packet speed	Question 4
QOE Test 1 2026 Q2.3	Polarization ellipse and special states	Question 5
QOE Test 1 2026 Q3.1	Dielectric wave, wavelength, B_0 , phase shift	Question 6
QOE Test 1 2026 Q3.2	Snell, Fresnel, reflectance, Brewster	Question 7
QOE Test 1 2026 Q3.3	Critical angle, acceptance angle, NA	Question 8
Tutorial 1	Propagation of light, Maxwell wave equation, dispersion, wave packets	Full question bank + Tutorial A
Tutorial 2	Plane-wave operators, field relations, refractive index, polarization	Full question bank + Tutorial B
Tutorial 3	Dielectric waves, Fresnel/Brewster, fibre optics	Full question bank + Tutorials C-D

Exam note

The worked sequence follows the same calculation-first pattern as the Test 2 pack. The question numbering above preserves the actual Test 1 paper grouping while presenting each major worked task separately.

TEST 2

LASERS & QUANTUM PHYSICS



CORE TOPICS

Laser Theory

Introduction to Quantum Physics

Schrodinger Equation and Applications

QOE580S TEST 2

STEP-BY-STEP CALCULATION PREPARATION

Quantum and Opto-Electronics 5

Strict scope only

Chapter 5: Laser Theory

Chapter 6: Introduction to Quantum Physics

Chapter 7: Schrodinger Equation and Applications

Calculation-first format

Complete Question → Diagram → Given → Find → Formula → Every Step → Boxed Final Answer

This pack was rebuilt from the supplied Test 2 material. It does not copy the structure of the source documents. Duplicate questions are merged into one full worked solution and mapped in the coverage index. Material outside the confirmed Test 2 scope, such as Tutorials 1-3, Fresnel/Brewster/fibre material and Fabry-Perot Chapter 4, is excluded.

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2.2	Chapter 6: Introduction to Quantum Physics	3
2.3	Chapter 7: Schrodinger Equation and Applications	3
3	Chapter 5 - Laser Theory: worked questions	4
3.1	Laser Theory rapid short-answer drill	11
4	Chapter 6 - Introduction to Quantum Physics: worked questions	12
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5	Chapter 7 - Schrodinger Equation and Applications: worked questions	19
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How to use this pack

The calculation style used throughout is intentionally simple:

1. Read the **complete question**.
2. Copy the **Given** values and write what must be found.
3. Write the **formula before substituting**.
4. Convert all units first.
5. Substitute one line at a time.
6. Keep units with numbers.
7. Show calculus, algebra, boundary conditions and reasons.
8. Box the final answer to 3 significant figures unless the question states otherwise.

Exam note

The older QOE Test 2 papers sometimes contain Chapter 4 Fabry-Perot and Brewster-angle questions. They are intentionally omitted here because you asked for **current Test 2 scope only**.

Compact formula sheet

Chapter 5: Laser Theory

$$\begin{aligned}
 R_{12} &= N_1 B_{12} \rho(\omega), \\
 R_{21}^{\text{sp}} &= N_2 A_{21}, \\
 R_{21}^{\text{st}} &= N_2 B_{21} \rho(\omega), \\
 \frac{N_2}{N_1} &= \exp \left[-\frac{E_2 - E_1}{k_B T} \right], \\
 A_{21} &= \frac{1}{\tau}, \\
 B_{12} &= B_{21} = B \quad (\text{equal degeneracy}), \\
 \frac{A}{B} &= \frac{8\pi h \nu^3}{c^3}, \\
 \frac{dN_2}{dt} &= N B \rho - (A + 2B \rho) N_2, \\
 N_2(t) &= \frac{N B \rho}{A + 2B \rho} \left(1 - e^{-(A+2B\rho)t} \right), \\
 \alpha &= \frac{\lambda^2}{8\pi \Delta \nu} (N_2 - N_1) A, \\
 I(z) &= I_0 e^{\alpha z}, \\
 \Delta \nu &\simeq \frac{c}{\lambda^2} \Delta \lambda, \\
 \alpha_{\text{th}} &= \gamma - \frac{1}{2L} \ln(R_1 R_2), \\
 R &= \left(\frac{n-1}{n+1} \right)^2, \\
 v_x &= \sqrt{\frac{k_B T}{M}}, \quad v_{\text{rms}} = \sqrt{\frac{3k_B T}{M}}, \\
 \Delta \nu_{\text{Doppler}} &\simeq \frac{2v}{c} \nu_0 = \frac{2v}{\lambda_0}.
 \end{aligned}$$

Chapter 6: Introduction to Quantum Physics

$$\begin{aligned}
 P &= \sigma A e T^4, & \lambda_{\max} T &= 2.898 \times 10^{-3} \text{ m K}, \\
 E &= hf = \frac{hc}{\lambda}, & p &= \frac{E}{c} = \frac{h}{\lambda}, \\
 K_{\max} &= hf - \phi, & hf_c &= \phi, \\
 eV_s &= K_{\max}, \\
 \lambda' &= \lambda + \frac{h}{m_e c} (1 - \cos \theta), \\
 \lambda_{\text{dB}} &= \frac{h}{p}, & p &= \sqrt{2m_e eV}, \\
 \Delta x \Delta p &\geq \frac{\hbar}{2}, \\
 mvr &= n\hbar, & r_n &= n^2 a_0, \\
 E_n &= -\frac{13.6}{n^2} \text{ eV}, \\
 \frac{1}{\lambda} &= R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right).
 \end{aligned}$$

Chapter 7: Schrodinger Equation and Applications

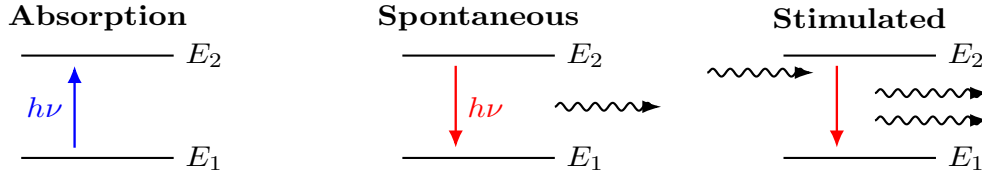
$$\begin{aligned}
 -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi &= E\psi, \\
 \int |\psi|^2 dV &= 1, \\
 \langle A \rangle &= \int \psi^* \hat{A} \psi dV, \\
 \psi_n(x) &= \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, \\
 E_n &= \frac{n^2 \pi^2 \hbar^2}{2mL^2} = \frac{n^2 h^2}{8mL^2}, \\
 \kappa &= \frac{\sqrt{2m(V_0 - E)}}{\hbar}, \\
 T &\approx e^{-2\kappa L}, \\
 V_{\text{HO}}(x) &= \frac{1}{2} m \omega_0^2 x^2, \\
 E_n^{\text{HO}} &= \left(n + \frac{1}{2} \right) \hbar \omega_0.
 \end{aligned}$$

Chapter 5 - Laser Theory: worked questions

Question 1 - Three Einstein light-atom processes

Source: QOE Test 2 2023 Q6; Laser Theory notes; GA Assignment 1

Complete question. Explain, with energy-level diagrams and rate expressions, (a) stimulated absorption, (b) spontaneous emission and (c) stimulated emission for a two-level atom.



(a) Stimulated absorption

$$h\nu = E_2 - E_1, \quad R_{12} = N_1 B_{12} \rho(\omega).$$

An incident photon is absorbed and the atom moves from E_1 to E_2 .

(b) Spontaneous emission

$$R_{21}^{\text{sp}} = N_2 A_{21}, \quad \tau = \frac{1}{A_{21}}.$$

No incident photon is required. The emitted photon has random direction and phase.

(c) Stimulated emission

$$R_{21}^{\text{st}} = N_2 B_{21} \rho(\omega).$$

An incident photon triggers the excited atom to emit a second photon with the same frequency, phase, direction and polarization.

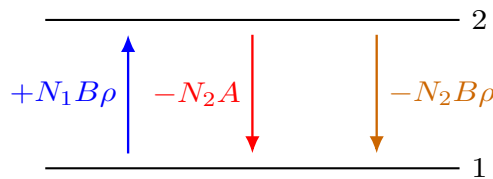
Final answer

Stimulated emission produces the coherent photon copy required for laser amplification.

Question 2 - Derive the upper-state rate equation

Source: Tutorial 4 Q1; Laser Theory Example 5.1; QOE Test 2 2023 Q7

Complete question. Derive the rate equation for $N_2(t)$ in a closed two-level system exposed to radiation of energy density $\rho(\omega_{21})$, assuming $B_{12} = B_{21} = B$ and $A_{21} = A$.



Step 1. Write all processes that change N_2 :

$$\frac{dN_2}{dt} = \underbrace{N_1 B \rho}_{\text{absorption in}} - \underbrace{N_2 B \rho}_{\text{stimulated out}} - \underbrace{N_2 A}_{\text{spontaneous out}}.$$

Step 2. Closed system means

$$N = N_1 + N_2 \quad \Rightarrow \quad N_1 = N - N_2.$$

Step 3. Substitute:

$$\begin{aligned} \frac{dN_2}{dt} &= (N - N_2) B \rho - N_2 B \rho - N_2 A \\ &= N B \rho - N_2 B \rho - N_2 B \rho - N_2 A \\ &= N B \rho - (A + 2B \rho) N_2. \end{aligned}$$

Final answer

$$\frac{dN_2}{dt} = NB\rho - (A + 2B\rho)N_2$$

Question 3 - Solve the rate equation for $N_2(0) = 0$

Source: *Laser Theory Exercise 5.2; QOE Test 2 2023 Q7*

Complete question. Solve

$$\frac{dN_2}{dt} + (A + 2B\rho)N_2 = NB\rho,$$

for $N_2(0) = 0$.

Step 1. Define

$$k = A + 2B\rho.$$

Then

$$\frac{dN_2}{dt} + kN_2 = NB\rho.$$

Step 2. Integrating factor:

$$\mu(t) = e^{\int k dt} = e^{kt}.$$

Step 3. Multiply the equation by e^{kt} :

$$e^{kt} \frac{dN_2}{dt} + ke^{kt} N_2 = NB\rho e^{kt}.$$

The left side is

$$\frac{d}{dt} (N_2 e^{kt}) = NB\rho e^{kt}.$$

Step 4. Integrate:

$$N_2 e^{kt} = NB\rho \int e^{kt} dt + C = \frac{NB\rho}{k} e^{kt} + C.$$

So

$$N_2 = \frac{NB\rho}{k} + C e^{-kt}.$$

Step 5. Apply $N_2(0) = 0$:

$$0 = \frac{NB\rho}{k} + C \Rightarrow C = -\frac{NB\rho}{k}.$$

Final answer

$$N_2(t) = \frac{NB\rho}{A + 2B\rho} \left(1 - e^{-(A+2B\rho)t} \right)$$

At long time,

$$N_2(\infty) = \frac{NB\rho}{A + 2B\rho} < \frac{N}{2}.$$

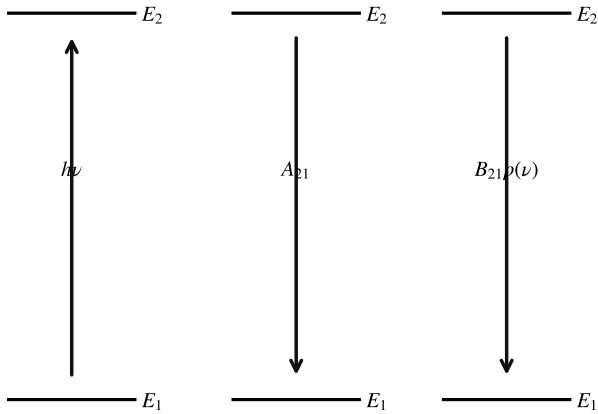
Therefore a two-level system cannot produce population inversion.

Question 4 - Steady-state upper-level population

Source: *Tutorial 4 Q2*

Complete question. A closed two-level system has $N = 1.0 \times 10^{20} \text{ m}^{-3}$, $B = 2.0 \times 10^{18} \text{ m}^3/(\text{J s}^2)$, $A = 1.0 \times 10^7 \text{ s}^{-1}$ and $\rho = 5.0 \times 10^{-12} \text{ J s/m}^3$. Find the steady-state N_2 .

Q1. Three Einstein light-atom processes

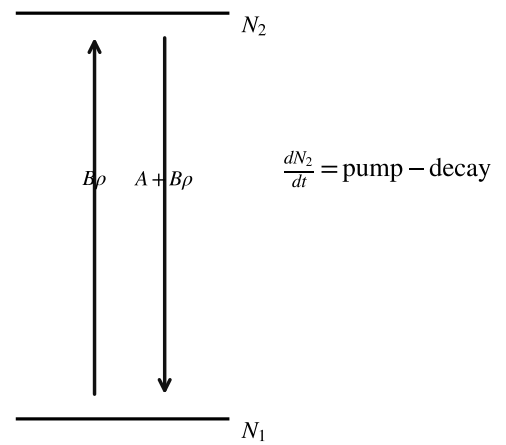


Absorption

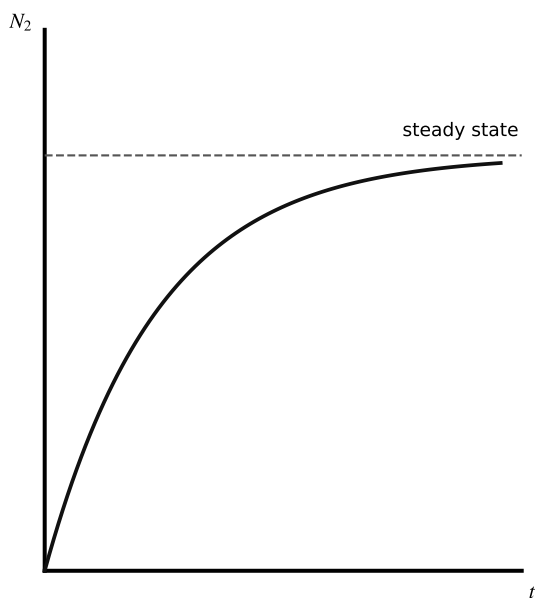
Spontaneous

Stimulated

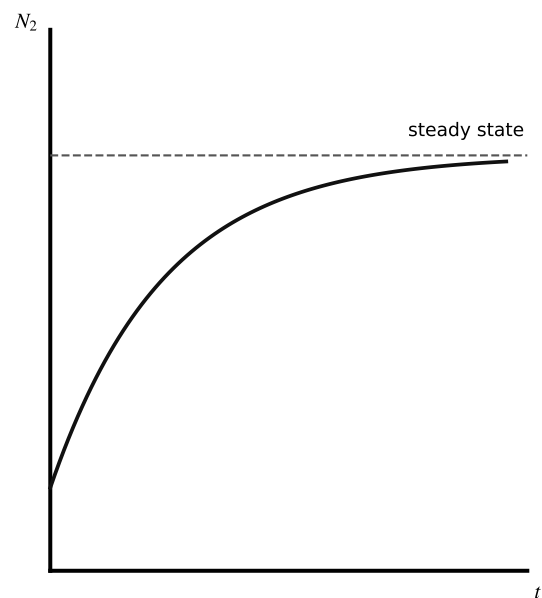
Q2. Derive the upper-state rate equation



Q3. Solve the rate equation for $N_2(0)=0$



Q4. Steady-state upper-level population



Given / Find

$$N = 1.0 \times 10^{20} \text{ m}^{-3}, \quad B = 2.0 \times 10^{18}, \quad A = 1.0 \times 10^7 \text{ s}^{-1}, \quad \rho = 5.0 \times 10^{-12}.$$

Find N_2 when $dN_2/dt = 0$.

Formula

$$N_{2,ss} = \frac{B\rho N}{2B\rho + A}.$$

Step 1. Compute $B\rho$:

$$B\rho = (2.0 \times 10^{18})(5.0 \times 10^{-12}) = 1.0 \times 10^7 \text{ s}^{-1}.$$

Step 2. Substitute:

$$N_2 = \frac{(1.0 \times 10^7)(1.0 \times 10^{20})}{2(1.0 \times 10^7) + 1.0 \times 10^7}.$$

Step 3. Simplify:

$$N_2 = \frac{1.0 \times 10^{27}}{3.0 \times 10^7} = 3.33 \times 10^{19} \text{ m}^{-3}.$$

Final answer

$$N_2 = 3.33 \times 10^{19} \text{ m}^{-3}$$

Question 5 - Laser gain coefficient

Source: Tutorial 4 Q3

Complete question. A gain medium operates at $\lambda = 632.8 \text{ nm}$ with $A = 3.1 \times 10^7 \text{ s}^{-1}$, $\Delta\nu = 1.5 \text{ GHz}$, $N_1 = 2.0 \times 10^{15} \text{ m}^{-3}$ and $N_2 = 8.0 \times 10^{15} \text{ m}^{-3}$. Calculate α .

Formula

$$\alpha = \frac{\lambda^2 A}{8\pi\Delta\nu}(N_2 - N_1).$$

Step 1. Convert units:

$$\lambda = 632.8 \times 10^{-9} = 6.328 \times 10^{-7} \text{ m}, \quad \Delta\nu = 1.5 \times 10^9 \text{ Hz}.$$

Step 2. Population inversion:

$$N_2 - N_1 = (8.0 - 2.0) \times 10^{15} = 6.0 \times 10^{15} \text{ m}^{-3}.$$

Step 3. Substitute:

$$\alpha = \frac{(6.328 \times 10^{-7})^2 (3.1 \times 10^7)}{8\pi(1.5 \times 10^9)} (6.0 \times 10^{15}).$$

Step 4. Calculate:

$$\begin{aligned} (6.328 \times 10^{-7})^2 &= 4.004 \times 10^{-13}. \\ \frac{(4.004 \times 10^{-13})(3.1 \times 10^7)}{8\pi(1.5 \times 10^9)} &= 3.293 \times 10^{-16} \text{ m}^2. \\ \alpha &= (3.293 \times 10^{-16})(6.0 \times 10^{15}) = 1.98 \text{ m}^{-1}. \end{aligned}$$

Final answer

$$\alpha = 1.98 \text{ m}^{-1}$$

Question 6 - Thermal population ratio and required temperature

Source: Tutorial 4 Q4

Complete question. A two-level gas cell has transition wavelength $\lambda = 1.06 \mu\text{m}$. (a) Find N_2/N_1 at 300 K. (b) Find the temperature required for $N_2/N_1 = 0.1$.

Formula

$$\frac{N_2}{N_1} = e^{-\Delta E/(k_B T)}, \quad \Delta E = \frac{hc}{\lambda}.$$

Step 1. Convert wavelength:

$$\lambda = 1.06 \times 10^{-6} \text{ m}.$$

Step 2. Transition energy:

$$\Delta E = \frac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{1.06 \times 10^{-6}} = 1.875 \times 10^{-19} \text{ J}.$$

Step 3. Ratio at 300 K:

$$k_B T = (1.38 \times 10^{-23})(300) = 4.14 \times 10^{-21} \text{ J}.$$

$$\frac{N_2}{N_1} = e^{-1.875 \times 10^{-19} / 4.14 \times 10^{-21}} = e^{-45.29} = 2.13 \times 10^{-20}.$$

Step 4. Required temperature for ratio 0.1:

$$0.1 = e^{-\Delta E/(k_B T)}.$$

Take natural logs:

$$\ln(0.1) = -\frac{\Delta E}{k_B T}.$$

Since $\ln(0.1) = -2.3026$,

$$T = \frac{\Delta E}{k_B (2.3026)} = \frac{1.875 \times 10^{-19}}{(1.38 \times 10^{-23})(2.3026)} = 5.90 \times 10^3 \text{ K}.$$

Final answer

$$\frac{N_2}{N_1} = 2.13 \times 10^{-20} \text{ at } 300 \text{ K},$$

$$T = 5.90 \times 10^3 \text{ K for } N_2/N_1 = 0.1.$$

Question 7 - Ruby laser transition and population ratios

Source: Tutorial 4 Q5

Complete question. A ruby laser emits at 694.3 nm. (a) Find the transition energy in joules and electron-volts. (b) Find the thermal-equilibrium population ratio at 27°C and 427°C.

Step 1. Convert wavelength:

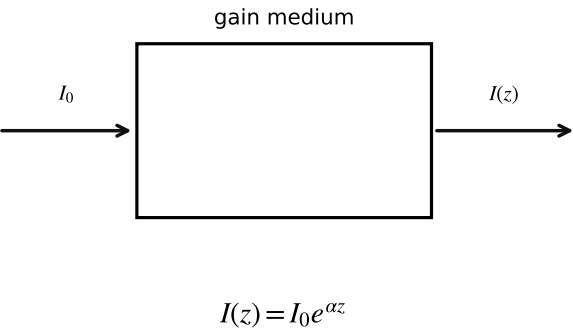
$$\lambda = 694.3 \times 10^{-9} = 6.943 \times 10^{-7} \text{ m}.$$

Step 2. Energy:

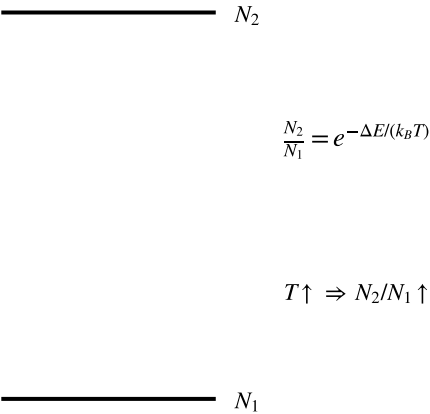
$$\Delta E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{6.943 \times 10^{-7}} = 2.863 \times 10^{-19} \text{ J}.$$

$$\Delta E = \frac{2.863 \times 10^{-19}}{1.602 \times 10^{-19}} = 1.79 \text{ eV}.$$

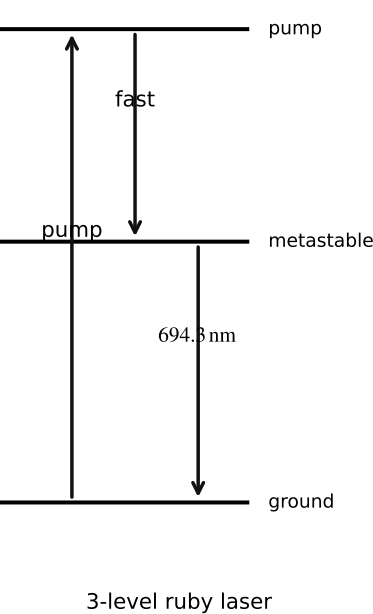
Q5. Laser gain coefficient



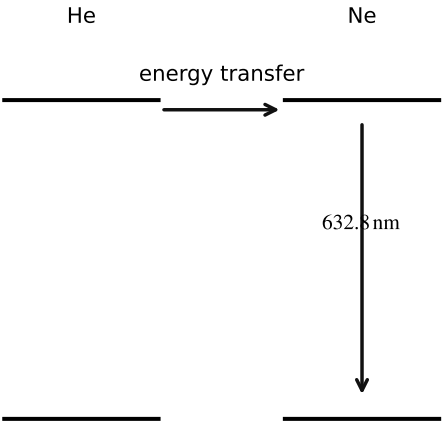
Q6. Thermal population ratio and required temperature



Q7. Ruby laser transition and population ratios



Q8. He-Ne transition and thermal population ratio



Step 3. At $T_1 = 27^\circ\text{C} = 300.15\text{ K}$:

$$\frac{N_2}{N_1} = \exp \left[-\frac{2.863 \times 10^{-19}}{(1.38 \times 10^{-23})(300.15)} \right] = e^{-69.12} \approx 9.6 \times 10^{-31}.$$

Step 4. At $T_2 = 427^\circ\text{C} = 700.15\text{ K}$:

$$\frac{N_2}{N_1} = \exp \left[-\frac{2.863 \times 10^{-19}}{(1.38 \times 10^{-23})(700.15)} \right] = e^{-29.63} = 1.35 \times 10^{-13}.$$

Final answer

$$\Delta E = 2.86 \times 10^{-19}\text{ J} = 1.79\text{ eV}$$

$$N_2/N_1 \approx 9.6 \times 10^{-31} \text{ at } 27^\circ\text{C}, \quad N_2/N_1 \approx 1.35 \times 10^{-13} \text{ at } 427^\circ\text{C}.$$

Question 8 - He-Ne transition and thermal population ratio

Source: Tutorial 4 Q6

Complete question. A He-Ne laser emits at 632.8 nm at 27°C . Find (a) the transition energy in joules and (b) the thermal-equilibrium ratio N_2/N_1 .

$$\lambda = 6.328 \times 10^{-7}\text{ m}, \quad T = 300.15\text{ K}.$$

$$\Delta E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{6.328 \times 10^{-7}} = 3.14 \times 10^{-19}\text{ J}.$$

$$\frac{N_2}{N_1} = \exp \left[-\frac{3.14 \times 10^{-19}}{(1.38 \times 10^{-23})(300.15)} \right] = e^{-75.8} \approx 1.2 \times 10^{-33}.$$

Final answer

$$\Delta E = 3.14 \times 10^{-19}\text{ J}, \quad N_2/N_1 \approx 1.2 \times 10^{-33}.$$

Question 9 - Derive the thermal rms speed

Source: QOE Test 2 2023 Q4

Complete question. Using first principles, derive the rms molecular speed in terms of T and M .

Step 1. Equipartition gives average translational kinetic energy:

$$\langle K \rangle = \frac{3}{2}k_B T.$$

Step 2. Define rms speed by

$$\langle K \rangle = \frac{1}{2}Mv_{\text{rms}}^2.$$

Step 3. Equate:

$$\frac{1}{2}Mv_{\text{rms}}^2 = \frac{3}{2}k_B T.$$

Multiply by 2:

$$Mv_{\text{rms}}^2 = 3k_B T.$$

Divide by M :

$$v_{\text{rms}}^2 = \frac{3k_B T}{M}.$$

Take the positive square root:

Final answer

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{M}}$$

For the lecturer's one-dimensional Doppler estimate use $v_x = \sqrt{k_B T / M}$ when that form is requested.

Question 10 - CO₂ laser Doppler spread

Source: QOE Test 2 2023 Q5

Complete question. A CO₂ laser operates at $T = 450$ K and $\lambda_0 = 10.0$ μm . Estimate the Doppler frequency spread. Use $M = 44u$ and the lecturer's one-dimensional speed.

Step 1. Molecular mass:

$$M = 44(1.66 \times 10^{-27}) = 7.30 \times 10^{-26} \text{ kg}.$$

Step 2. Thermal speed:

$$v = \sqrt{\frac{k_B T}{M}} = \sqrt{\frac{(1.38 \times 10^{-23})(450)}{7.30 \times 10^{-26}}} = 292 \text{ m/s}.$$

Step 3. Central frequency:

$$\nu_0 = \frac{c}{\lambda_0} = \frac{3.00 \times 10^8}{10.0 \times 10^{-6}} = 3.00 \times 10^{13} \text{ Hz}.$$

Step 4. Doppler spread:

$$\Delta\nu \simeq \frac{2v}{c} \nu_0 = \frac{2(292)}{3.00 \times 10^8} (3.00 \times 10^{13}) = 5.83 \times 10^7 \text{ Hz}.$$

Final answer

$$\Delta\nu \approx 5.83 \times 10^7 \text{ Hz} = 58.3 \text{ MHz}$$

Question 11 - Gain constant from wavelength linewidth

Source: QOE Test 2 2024 Q2

Complete question. A laser has inversion density 10^{17} cm^{-3} , wavelength 750 nm, linewidth 1.00 nm and spontaneous lifetime 10^{-4} s . Calculate the gain constant.

Step 1. Density conversion:

$$10^{17} \text{ cm}^{-3} = 10^{17} \times 10^6 = 1.0 \times 10^{23} \text{ m}^{-3}.$$

Step 2. Spontaneous coefficient:

$$A = \frac{1}{\tau} = \frac{1}{10^{-4}} = 1.0 \times 10^4 \text{ s}^{-1}.$$

Step 3. Convert linewidth from wavelength to frequency:

$$\Delta\nu \simeq \frac{c}{\lambda^2} \Delta\lambda.$$

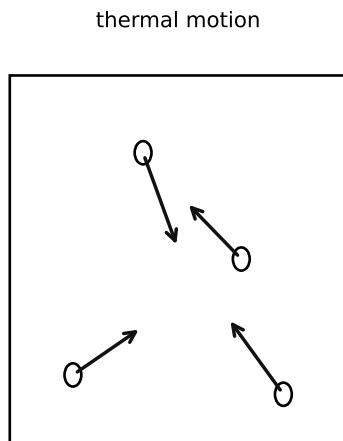
$$\Delta\nu = \frac{3.00 \times 10^8}{(750 \times 10^{-9})^2} (1.00 \times 10^{-9}) = 5.33 \times 10^{11} \text{ Hz}.$$

Step 4. Gain:

$$\alpha = \frac{(750 \times 10^{-9})^2}{8\pi(5.33 \times 10^{11})} (1.0 \times 10^{23})(1.0 \times 10^4).$$

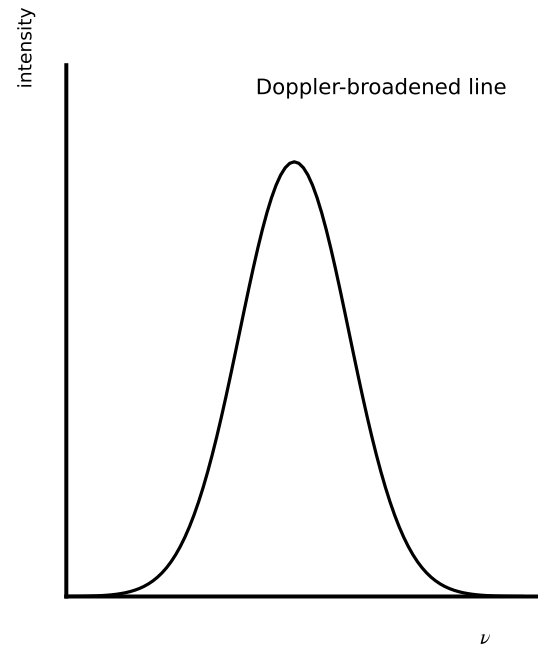
$$\alpha = 42.0 \text{ m}^{-1}.$$

Q9. Derive the thermal rms speed

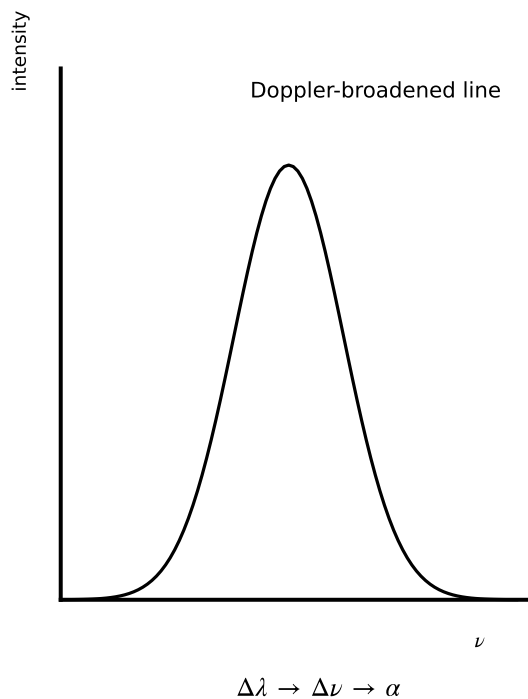


$$v_{\text{rms}} = \sqrt{3k_B T/M}$$

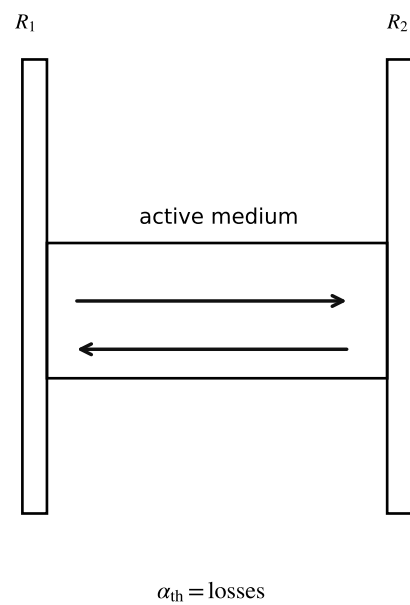
Q10. CO₂ laser Doppler spread



Q11. Gain constant from wavelength linewidth



Q12. GaAs laser threshold gain



Final answer

$$\alpha = 42.0 \text{ m}^{-1}$$

Question 12 - GaAs laser threshold gain

Source: QOE Test 2 2024 Q3

Complete question. A GaAs laser diode has cavity length $L = 500 \text{ } \mu\text{m}$, refractive index $n = 3.6$ and internal loss $\gamma = 10 \text{ cm}^{-1}$. Using cleaved-facet reflectance, find the threshold gain.

Step 1. Facet reflectance:

$$R = \left(\frac{n-1}{n+1} \right)^2 = \left(\frac{3.6-1}{3.6+1} \right)^2 = \left(\frac{2.6}{4.6} \right)^2 = 0.319.$$

Thus $R_1 = R_2 = 0.319$.

Step 2. Convert units:

$$L = 500 \times 10^{-6} = 5.00 \times 10^{-4} \text{ m},$$

$$\gamma = 10 \text{ cm}^{-1} = 10(100) = 1000 \text{ m}^{-1}.$$

Step 3. Threshold formula:

$$\alpha_{\text{th}} = \gamma - \frac{1}{2L} \ln(R_1 R_2).$$

$$\alpha_{\text{th}} = 1000 - \frac{1}{2(5.00 \times 10^{-4})} \ln[(0.319)(0.319)].$$

$$\ln(0.319^2) \approx -2.282.$$

$$\alpha_{\text{th}} = 1000 - 1000(-2.282) = 3.28 \times 10^3 \text{ m}^{-1}.$$

Final answer

$$\alpha_{\text{th}} = 3.28 \times 10^3 \text{ m}^{-1} = 32.8 \text{ cm}^{-1}$$

Question 13 - Derive the Einstein A-B relation

Source: GA Assignment 1 Q15; Laser Theory notes

Complete question. Derive Einstein's relation between spontaneous and stimulated emission coefficients for non-degenerate levels.

At thermal equilibrium,

$$N_1 B_{12} \rho = N_2 A_{21} + N_2 B_{21} \rho.$$

Move the ρ terms together:

$$\rho(N_1 B_{12} - N_2 B_{21}) = N_2 A_{21}.$$

Divide by N_2 :

$$\rho = \frac{A_{21}}{(N_1/N_2)B_{12} - B_{21}}.$$

Use Boltzmann:

$$\frac{N_1}{N_2} = e^{h\nu/(k_B T)}.$$

Therefore

$$\rho = \frac{A_{21}}{B_{12} e^{h\nu/(k_B T)} - B_{21}}.$$

Planck's law is

$$\rho = \frac{8\pi h \nu^3 / c^3}{e^{h\nu/(k_B T)} - 1}.$$

To match for every temperature:

$$B_{12} = B_{21} = B,$$

and

$$\frac{A_{21}}{B} = \frac{8\pi h\nu^3}{c^3}.$$

Final answer

$$B_{12} = B_{21}, \quad \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}.$$

Question 14 - Exercise: what does degenerate mean?

Source: Laser Theory Exercise 5.1 Two or more different quantum states are **degenerate** when they have the same energy eigenvalue.

Final answer

Example: in a square 2D box, states $(n_x, n_y) = (1, 2)$ and $(2, 1)$ have the same energy.

Laser Theory rapid short-answer drill

These are the Test 2-relevant GA Assignment 1 theory questions. Keep answers short in the test.

#	Question	Short answer
1	Distinguish ordinary and laser light.	Ordinary: incoherent, broad spectrum, divergent, low intensity. Laser: coherent, nearly monochromatic, directional, high intensity.
2	Distinguish spontaneous and stimulated emission.	Spontaneous needs no incoming photon and has random phase/direction. Stimulated emission is triggered by a matching photon and produces a coherent copy.
3	Characteristics of LASER.	Coherent, monochromatic, highly directional, high brightness/intensity.
4	What is spontaneous emission?	Excited atom decays by itself: rate $N_2 A_{21}$.
5	What is stimulated emission?	Matching photon triggers excited atom to emit an identical photon: rate $N_2 B_{21} \rho$.
6	Population inversion?	$N_2 > N_1$, required for net gain.
7	Pumping?	Supplying energy to create and maintain inversion.
8	Pumping methods?	Optical, electrical discharge, current injection, chemical, thermal.
9	Conditions for laser action?	Active medium, metastable level, pumping/inversion, resonator feedback, gain exceeding loss.
10	Einstein coefficients?	A_{21} spontaneous; B_{12} absorption; B_{21} stimulated emission.
11	Active medium?	Material in which population inversion and optical gain occur.
12	Optical resonator?	Two-mirror cavity that provides feedback and mode selection; one mirror partly transmits output.
13	He-Ne laser construction/working.	He-Ne gas mixture, electrical discharge excites He, collisions transfer energy to Ne, four-level action, 632.8 nm output.
14	Ruby laser construction/working.	Cr^{3+} -doped Al_2O_3 rod, flashlamp pumping, three-level system, 694.3 nm pulsed output.
15	Einstein relation.	Detailed derivation is Question 13 above.
16	Laser applications.	Communication, surgery, cutting/welding, LIDAR, spectroscopy, barcode/CD/DVD, alignment, holography.

Chapter 6 - Introduction to Quantum Physics: worked questions

Question 15 - Blackbody radiation around you

Source: QM Practice Questions 2025, Section C

Complete question. Use Wien's law to analyse: (a) a human at 310 K; (b) a stove whose peak wavelength is 600 nm; (c) cosmic background radiation at 2.7 K.

Formula

$$\lambda_{\max} T = b, \quad b = 2.898 \times 10^{-3} \text{ m K}, \quad f = \frac{c}{\lambda}.$$

(a) **Human**, $T = 310 \text{ K}$

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}}{310} = 9.35 \times 10^{-6} \text{ m} = 9.35 \text{ } \mu\text{m}.$$

$$f = \frac{3.00 \times 10^8}{9.35 \times 10^{-6}} = 3.21 \times 10^{13} \text{ Hz}.$$

This is infrared.

(b) **Stove**, $\lambda_{\max} = 600 \text{ nm}$

$$600 \text{ nm} = 6.00 \times 10^{-7} \text{ m}.$$

$$T = \frac{2.898 \times 10^{-3}}{6.00 \times 10^{-7}} = 4.83 \times 10^3 \text{ K}.$$

600 nm is in the orange-red visible region.

(c) **Cosmic background**, $T = 2.7 \text{ K}$

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}}{2.7} = 1.07 \times 10^{-3} \text{ m} = 1.07 \text{ mm}.$$

$$f = \frac{3.00 \times 10^8}{1.07 \times 10^{-3}} = 2.80 \times 10^{11} \text{ Hz}.$$

This is microwave radiation.

Final answer

Human: $9.35 \text{ } \mu\text{m}$, IR. Stove: $4.83 \times 10^3 \text{ K}$. CMB: 1.07 mm , microwave.

Question 16 - Sun as a blackbody

Source: Physics 5 Test 2 2024 Q1

Complete question. The Sun has radius $R = 6.96 \times 10^8 \text{ m}$ and total power $P = 3.85 \times 10^{26} \text{ W}$. Assuming blackbody emission, find (a) surface temperature and (b) $\lambda_{\max}$.

Step 1. Surface area:

$$A = 4\pi R^2 = 4\pi(6.96 \times 10^8)^2.$$

Step 2. Stefan-Boltzmann law:

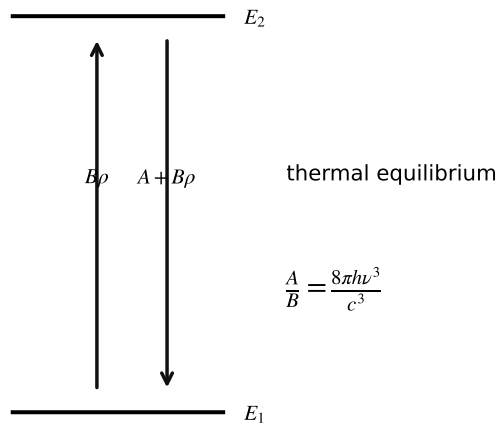
$$P = \sigma AT^4 \Rightarrow T = \left(\frac{P}{\sigma 4\pi R^2} \right)^{1/4}.$$

$$T = \left[\frac{3.85 \times 10^{26}}{(5.67 \times 10^{-8}) 4\pi(6.96 \times 10^8)^2} \right]^{1/4} = 5.78 \times 10^3 \text{ K}.$$

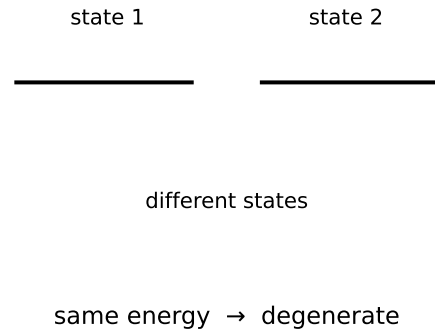
Step 3. Wien's law:

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}}{5.78 \times 10^3} = 5.01 \times 10^{-7} \text{ m} = 501 \text{ nm}.$$

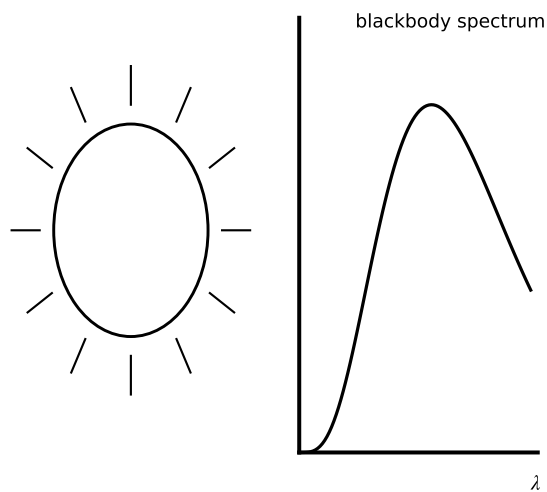
Q13. Derive the Einstein A-B relation



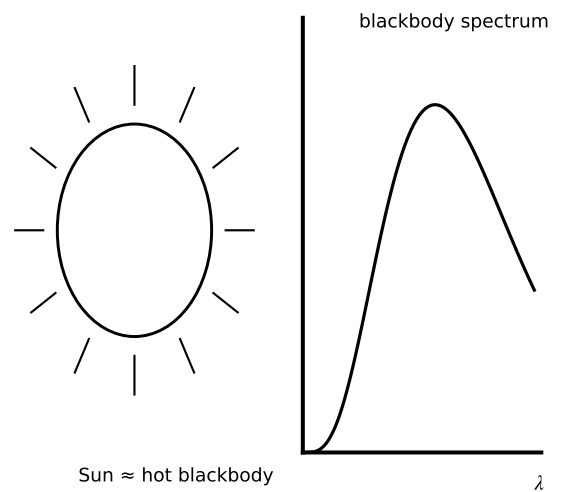
Q14. Exercise: what does degenerate mean?



Q15. Blackbody radiation around you



Q16. Sun as a blackbody



Final answer

$$T \approx 5.78 \times 10^3 \text{ K}, \quad \lambda_{\max} \approx 501 \text{ nm}.$$

Question 17 - Photon momentum at three energies/frequencies

Source: Assignment 1 Atomic and Quantum Physics Q1

Complete question. Find the photon momentum for (a) a 10.0 MeV gamma ray, (b) a $1.0 \mu\text{m}$ infrared photon and (c) a 150 MHz radio photon. Give kg m/s and eV/c.

Formula

$$p = \frac{E}{c} = \frac{h}{\lambda} = \frac{hf}{c}.$$

(a) 10.0 MeV

$$E = (10.0 \times 10^6)(1.602 \times 10^{-19}) = 1.602 \times 10^{-12} \text{ J}.$$

$$p = \frac{1.602 \times 10^{-12}}{3.00 \times 10^8} = 5.34 \times 10^{-21} \text{ kg m/s}.$$

In eV/c: $p = 10.0 \text{ MeV}/c$.

(b) $\lambda = 1.0 \mu\text{m} = 1.0 \times 10^{-6} \text{ m}$

$$p = \frac{6.626 \times 10^{-34}}{1.0 \times 10^{-6}} = 6.63 \times 10^{-28} \text{ kg m/s}.$$

Photon energy is

$$E = \frac{hc}{\lambda} = 1.24 \text{ eV},$$

so $p = 1.24 \text{ eV}/c$.

(c) $f = 150 \text{ MHz} = 1.50 \times 10^8 \text{ Hz}$

$$p = \frac{hf}{c} = \frac{(6.626 \times 10^{-34})(1.50 \times 10^8)}{3.00 \times 10^8} = 3.31 \times 10^{-34} \text{ kg m/s}.$$

$$E = hf = 6.20 \times 10^{-7} \text{ eV} \Rightarrow p = 6.20 \times 10^{-7} \text{ eV}/c.$$

Final answer

(a) $5.34 \times 10^{-21} \text{ kg m/s}$; (b) $6.63 \times 10^{-28} \text{ kg m/s}$; (c) $3.31 \times 10^{-34} \text{ kg m/s}$.

Question 18 - Photoelectric cutoff wavelength

Source: Assignment 1 Atomic and Quantum Physics Q2

Complete question. The cutoff wavelength for a metal is 254 nm. (a) Find the work function. (b) Will photoemission occur for $\lambda > 254 \text{ nm}$ or $\lambda < 254 \text{ nm}$?

At cutoff, $K_{\max} = 0$:

$$\phi = hf_c = \frac{hc}{\lambda_c}.$$

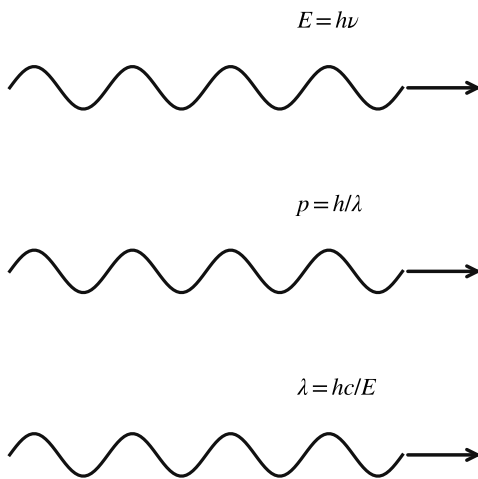
$$\phi = \frac{1242 \text{ eV nm}}{254 \text{ nm}} = 4.89 \text{ eV}.$$

Photoemission requires $hf > \phi$, so it requires higher frequency and therefore shorter wavelength:

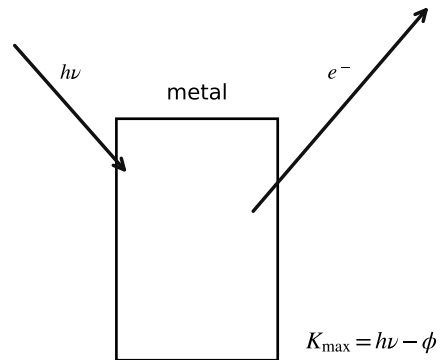
Final answer

$$\phi \approx 4.89 \text{ eV}, \quad \text{photoemission for } \lambda < 254 \text{ nm}.$$

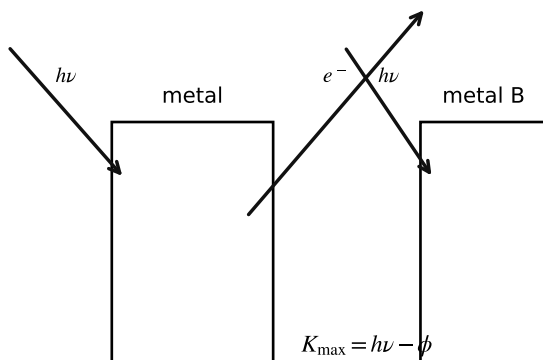
Q17. Photon momentum at three energies/frequencies



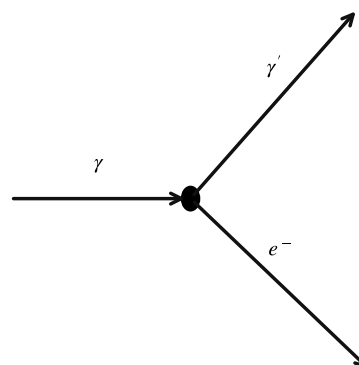
Q18. Photoelectric cutoff wavelength



Q19. Photoelectric effect with two metals



Q20. Compton scattering of a photon at 45°



Question 19 - Photoelectric effect with two metals

Source: Physics 5 Test 2 2024 Q3

Complete question. Metal A has $\phi_A = 4.7$ eV and gives $K_A = 0.8$ eV. Metal B under the same light gives $K_B = 3.2$ eV. Find (a) the wavelength and (b) ϕ_B .

Step 1. Photon energy from metal A:

$$hf = \phi_A + K_A = 4.7 + 0.8 = 5.5 \text{ eV}.$$

Step 2. Wavelength:

$$\lambda = \frac{hc}{E} = \frac{1242 \text{ eV nm}}{5.5 \text{ eV}} = 226 \text{ nm}.$$

Step 3. Work function of B:

$$K_B = hf - \phi_B \Rightarrow \phi_B = hf - K_B = 5.5 - 3.2 = 2.3 \text{ eV}.$$

Final answer

$$\lambda = 226 \text{ nm}, \quad \phi_B = 2.30 \text{ eV}.$$

Question 20 - Compton scattering of a 10.39 keV photon at 45°

Source: Assignment 1 Atomic and Quantum Physics Q4

Complete question. Incident photons have energy 10.39 keV and scatter through 45.00°. Find (a) the scattered photon energy and (b) the recoil-electron kinetic energy.

Step 1. Incident wavelength:

$$\lambda = \frac{hc}{E} = \frac{1242 \text{ eV nm}}{10.39 \times 10^3 \text{ eV}} = 0.1195 \text{ nm}.$$

Step 2. Compton shift:

$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos 45^\circ).$$

Using $h/(m_e c) = 0.002426$ nm:

$$\Delta\lambda = (0.002426)(1 - 0.7071) = 0.000711 \text{ nm}.$$

Step 3. Scattered wavelength:

$$\lambda' = 0.1195 + 0.000711 = 0.1202 \text{ nm}.$$

Step 4. Scattered energy:

$$E' = \frac{1242}{0.1202} = 1.033 \times 10^4 \text{ eV} = 10.33 \text{ keV}.$$

Step 5. Recoil kinetic energy:

$$K_e = E - E' = 10.39 - 10.33 = 0.0615 \text{ keV} = 61.5 \text{ eV}.$$

Final answer

$$E' \approx 10.33 \text{ keV}, \quad K_e \approx 61.5 \text{ eV}.$$

Question 21 - Compton wavelengths at 0°, 45° and 180°

Source: Assignment 1 Atomic and Quantum Physics Q5

Complete question. A photon of wavelength 0.0900 nm scatters from carbon. Find the scattered wavelength at 0° , 45° and 180° .

$$\lambda' = \lambda + \lambda_C(1 - \cos \theta), \quad \lambda_C = 0.002426 \text{ nm.}$$

At 0° :

$$\lambda' = 0.0900 + 0.002426(1 - 1) = 0.0900 \text{ nm.}$$

At 45° :

$$\lambda' = 0.0900 + 0.002426(1 - 0.7071) = 0.09071 \text{ nm.}$$

At 180° :

$$\lambda' = 0.0900 + 0.002426(1 - (-1)) = 0.09485 \text{ nm.}$$

Final answer

$$0.0900 \text{ nm, } 0.0907 \text{ nm, } 0.0949 \text{ nm}$$

Question 22 - Compton scattering at 60°

Source: Physics 5 Test 2 2023 Q1

Complete question. X-rays of wavelength 0.2400 nm scatter through 60.00° . Find (a) scattered wavelength, (b) scattered photon energy and (c) recoil-electron kinetic energy.

Step 1. Wavelength shift:

$$\lambda' = 0.2400 + 0.002426(1 - \cos 60^\circ) = 0.2400 + 0.001213 = 0.2412 \text{ nm.}$$

Step 2. Scattered energy:

$$E' = \frac{1242}{0.2412} = 5.15 \times 10^3 \text{ eV} = 5.15 \text{ keV.}$$

Step 3. Incident energy:

$$E = \frac{1242}{0.2400} = 5.175 \text{ keV.}$$

Step 4. Electron kinetic energy:

$$K_e = E - E' = 5.175 - 5.149 \approx 0.0260 \text{ keV} = 26.0 \text{ eV.}$$

Final answer

$$\lambda' = 0.2412 \text{ nm}, \quad E' = 5.15 \text{ keV}, \quad K_e = 26.0 \text{ eV}.$$

Question 23 - Compton event: find incident wavelength, photon angle and electron angle

Source: Physics 5 Test 2 2024 Q2

Complete question. The scattered photon has energy 135 keV and the recoil electron has kinetic energy 25 keV. Find (a) incident wavelength, (b) photon scattering angle, (c) electron recoil angle.

Step 1. Energy conservation:

$$E = E' + K_e = 135 + 25 = 160 \text{ keV.}$$

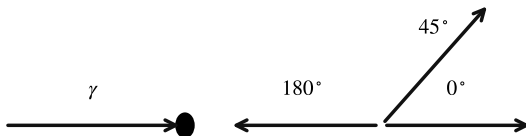
Step 2. Incident wavelength:

$$\lambda = \frac{1242 \text{ eV nm}}{160 \times 10^3 \text{ eV}} = 0.00776 \text{ nm} = 7.76 \text{ pm.}$$

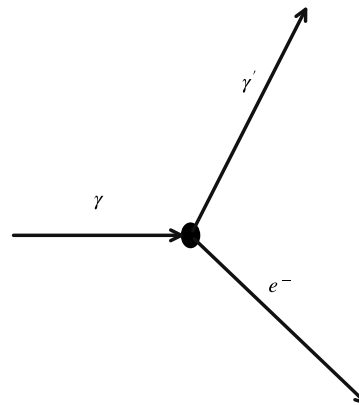
Step 3. Scattered wavelength:

$$\lambda' = \frac{1242}{135 \times 10^3} = 0.00920 \text{ nm} = 9.20 \text{ pm.}$$

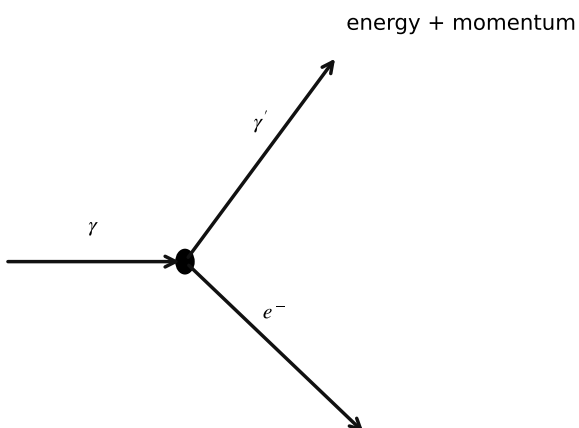
Q21. Compton wavelengths at $0^\circ, 45^\circ, 180^\circ$



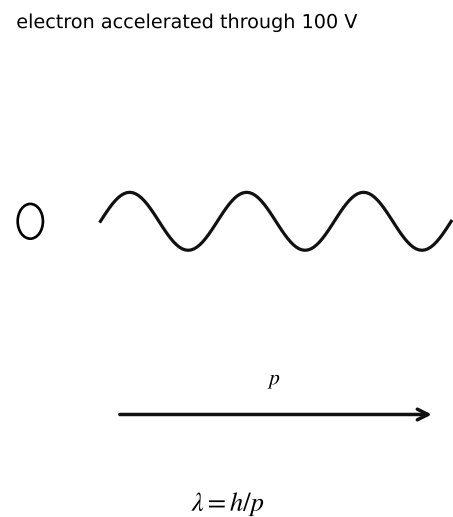
Q22. Compton scattering at 60°



Q23. Compton event: incident, photon and electron angles



Q24. de Broglie wavelength of a 100 V electron



Step 4. Photon angle:

$$\begin{aligned}\lambda' - \lambda &= \lambda_C(1 - \cos \theta). \\ 1 - \cos \theta &= \frac{9.20 - 7.76}{2.43} = 0.593. \\ \cos \theta &= 0.407 \quad \Rightarrow \quad \theta = 66.0^\circ.\end{aligned}$$

Step 5. Momentum components for electron angle ϕ :

$$\begin{aligned}\frac{h}{\lambda} &= \frac{h}{\lambda'} \cos \theta + p_e \cos \phi, \\ 0 &= \frac{h}{\lambda'} \sin \theta - p_e \sin \phi.\end{aligned}$$

Divide the y component by the electron x component:

$$\tan \phi = \frac{(1/\lambda') \sin \theta}{1/\lambda - (1/\lambda') \cos \theta}.$$

Substitution gives

$$\phi \approx 49.6^\circ.$$

Final answer

$$\boxed{\lambda = 7.76 \text{ pm}}, \quad \boxed{\theta = 66.0^\circ}, \quad \boxed{\phi = 49.6^\circ}.$$

Question 24 - de Broglie wavelength of a 100 V electron

Source: Chapter 6 worked example

Complete question. Find the de Broglie wavelength of an electron accelerated from rest through 100 V.

$$\begin{aligned}eV &= \frac{p^2}{2m_e} \quad \Rightarrow \quad p = \sqrt{2m_e eV}. \\ p &= \sqrt{2(9.11 \times 10^{-31})(1.602 \times 10^{-19})(100)} = 5.40 \times 10^{-24} \text{ kg m/s}. \\ \lambda &= \frac{h}{p} = \frac{6.626 \times 10^{-34}}{5.40 \times 10^{-24}} = 1.23 \times 10^{-10} \text{ m} = 0.123 \text{ nm}.\end{aligned}$$

Final answer

$$\boxed{\lambda = 0.123 \text{ nm}}$$

Question 25 - Derive Bohr angular-momentum quantization

Source: Chapter 6 notes / mock-style derivation

Complete question. Starting from the standing-wave condition $n\lambda = 2\pi r$ and de Broglie relation $\lambda = h/p$, show that orbital angular momentum is quantized.

$$n\lambda = 2\pi r.$$

Substitute $\lambda = h/p$:

$$n \frac{h}{p} = 2\pi r.$$

Multiply by p :

$$nh = 2\pi pr.$$

Therefore

$$pr = n \frac{h}{2\pi} = n\hbar.$$

For a circular orbit $p = mv$, so

Final answer

$$mvr = n\hbar$$

Question 26 - Hydrogen transitions and radius

Source: Chapter 6 worked example

Complete question. (a) Find the photon wavelength for $n = 3 \rightarrow 1$. (b) Find the Balmer H-alpha wavelength $3 \rightarrow 2$. (c) Find the radius for $n = 3$.

(a) $3 \rightarrow 1$:

$$\Delta E = 13.6 \left(1 - \frac{1}{9} \right) = 12.09 \text{ eV.}$$

$$\lambda = \frac{1242}{12.09} = 102.7 \text{ nm} \approx 103 \text{ nm.}$$

This is Lyman ultraviolet.

(b) $3 \rightarrow 2$:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{4} - \frac{1}{9} \right).$$

$$\lambda \approx 656 \text{ nm.}$$

This is Balmer H-alpha, red.

(c) **Radius:**

$$r_3 = n^2 a_0 = 9(0.0529 \text{ nm}) = 0.476 \text{ nm.}$$

Final answer

$$\lambda_{3 \rightarrow 1} \approx 103 \text{ nm}, \quad \lambda_{3 \rightarrow 2} \approx 656 \text{ nm}, \quad r_3 = 0.476 \text{ nm}.$$

Quantum-theory rapid questions

Question	Short answer
What is a blackbody?	Ideal absorber/emitter; emission spectrum depends only on temperature.
Why does a hot body radiate?	Thermally moving charges and atomic transitions emit EM radiation.
Which is hotter, red or blue glow?	Blue, because Wien's law gives shorter λ_{max} for higher T .
What was the ultraviolet catastrophe?	Classical Rayleigh-Jeans theory predicted infinite energy at short wavelength. Planck quantization removed the divergence.
Why can UV eject electrons when bright red cannot?	Photon energy is hf . If $hf < \phi$, no electron is ejected regardless of intensity.
What did photoelectric effect prove?	Light has particle-like quanta, photons.
What does Compton scattering prove?	Photons carry momentum $p = h/\lambda$ and exchange energy/momentum like particles.
Why was quantum mechanics introduced?	Classical physics failed for blackbody radiation, photoelectric effect, Compton scattering, atomic stability and line spectra.
Define photon.	Quantum of light with $E = hf$ and $p = h/\lambda$.
Define Planck's constant.	Constant linking energy and frequency: $E = hf$.
Define quantization.	Only discrete values are allowed for certain observables, such as atomic energies.

Question	Short answer
Wave-particle duality?	Light and matter can show both wave and particle behavior depending on the experiment.

Question 27 - Bonus Wien-law calculation at 300°C

Source: QM Practice Questions 2025 bonus

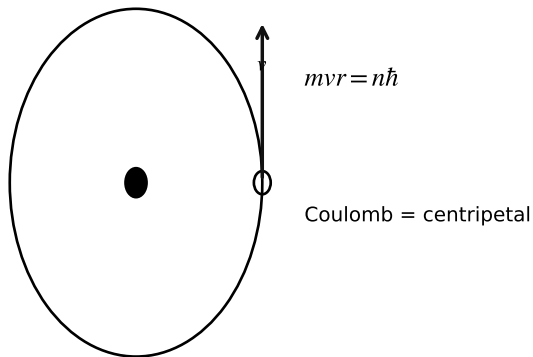
$$T = 300 + 273.15 = 573.15 \text{ K.}$$

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}}{573.15} = 5.06 \times 10^{-6} \text{ m.}$$

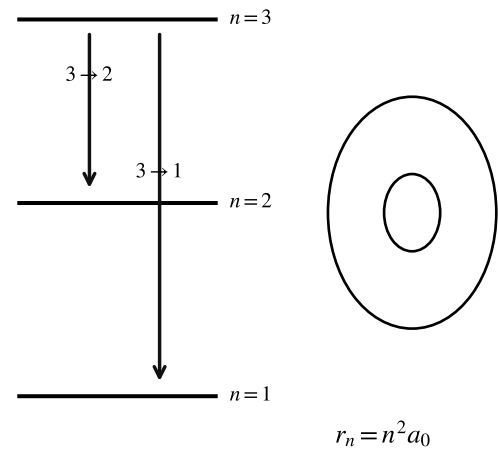
Final answer

$$\lambda_{\max} = 5.06 \text{ }\mu\text{m}$$

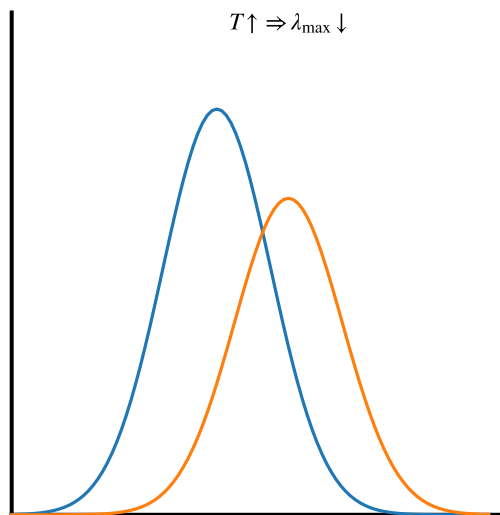
Q25. Derive Bohr angular-momentum quantization



Q26. Hydrogen transitions and radius



Q27. Bonus Wien-law calculation at 300°C



Q28. Separate the time-dependent Schrodinger equation

$$\Psi(r, t) = \psi(r)T(t)$$

space part

time part

Chapter 7 - Schrodinger Equation and Applications: worked questions

Question 28 - Separate the time-dependent Schrodinger equation

Source: Quantum Lecture 1

Complete question. For a time-independent potential, separate $\Psi(\mathbf{r}, t)$ into spatial and temporal parts and obtain the time-independent Schrodinger equation.

Start with

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}.$$

Assume

$$\Psi(\mathbf{r}, t) = \psi(\mathbf{r})T(t).$$

Substitute:

$$-\frac{\hbar^2}{2m}T\nabla^2\psi + V\psi T = i\hbar\psi\frac{dT}{dt}.$$

Divide by ψT :

$$\frac{1}{\psi} \left[-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \right] = \frac{i\hbar}{T} \frac{dT}{dt}.$$

The left side depends only on position and the right side only on time, so both equal a constant E .

Spatial equation:

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi}.$$

Time equation:

$$i\hbar\frac{dT}{dt} = ET.$$

Thus

$$\frac{dT}{T} = -\frac{iE}{\hbar}dt.$$

Integrate:

$$\ln T = -\frac{iEt}{\hbar} + C.$$

Hence

Final answer

$$\boxed{T(t) = e^{-iEt/\hbar}}, \quad \boxed{\Psi(\mathbf{r}, t) = \psi(\mathbf{r})e^{-iEt/\hbar}}.$$

Question 29 - Normalize a Gaussian wavefunction

Source: Some Solved Problems Q1

Complete question. A particle has $\psi(x) = Ae^{-ax^2}$. Find A if the wavefunction is normalized.

Normalization:

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1.$$

Since ψ is real,

$$|\psi|^2 = A^2 e^{-2ax^2}.$$

Therefore

$$A^2 \int_{-\infty}^{\infty} e^{-2ax^2} dx = 1.$$

Use

$$\int_{-\infty}^{\infty} e^{-bx^2} dx = \sqrt{\frac{\pi}{b}}.$$

With $b = 2a$:

$$A^2 \sqrt{\frac{\pi}{2a}} = 1.$$

Thus

$$A^2 = \sqrt{\frac{2a}{\pi}}.$$

Taking the positive real root,

Final answer

$$A = \left(\frac{2a}{\pi} \right)^{1/4}$$

Question 30 - Expectation value $\langle x \rangle$ in an infinite box

Source: Some Solved Problems Q2

Complete question. For a particle in $0 < x < L$ with

$$\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L},$$

find $\langle x \rangle$.

$$\langle x \rangle = \int_0^L \psi_n^* x \psi_n dx = \frac{2}{L} \int_0^L x \sin^2 \frac{n\pi x}{L} dx.$$

Use

$$\sin^2 u = \frac{1}{2}(1 - \cos 2u).$$

Then

$$\langle x \rangle = \frac{1}{L} \int_0^L x dx - \frac{1}{L} \int_0^L x \cos \frac{2n\pi x}{L} dx.$$

First integral:

$$\frac{1}{L} \left[\frac{x^2}{2} \right]_0^L = \frac{L}{2}.$$

For the second integral, let $a = 2n\pi/L$:

$$\int x \cos(ax) dx = \frac{x \sin(ax)}{a} + \frac{\cos(ax)}{a^2}.$$

At 0 and L , $\sin(2n\pi) = 0$ and $\cos(2n\pi) = \cos 0 = 1$, so the second integral is zero.

Final answer

$$\langle x \rangle = \frac{L}{2}$$

Question 31 - Verify a time-dependent box solution

Source: Some Solved Problems Q3

Complete question. Show that

$$\Psi(x, t) = A \sin(kx) e^{-iEt/\hbar}$$

solves the Schrodinger equation inside a well with $V = 0$ provided $E = \hbar^2 k^2 / (2m)$.

Time derivative:

$$\frac{\partial \Psi}{\partial t} = -\frac{iE}{\hbar} \Psi.$$

Therefore

$$i\hbar \frac{\partial \Psi}{\partial t} = E\Psi.$$

Spatial second derivative:

$$\frac{\partial^2 \Psi}{\partial x^2} = -k^2 \Psi.$$

Thus the kinetic side is

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} = \frac{\hbar^2 k^2}{2m} \Psi.$$

For both sides to be equal,

Final answer

$$E = \frac{\hbar^2 k^2}{2m}.$$

Question 32 - Solve the infinite one-dimensional well

Source: Some Solved Problems Q4

Complete question. A particle of mass m is trapped in $0 \leq x \leq a$ with $V = 0$ inside and $V = \infty$ outside. Solve the Schrodinger equation.

Inside the well:

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E\psi.$$

Rearrange:

$$\frac{d^2 \psi}{dx^2} + k^2 \psi = 0, \quad k^2 = \frac{2mE}{\hbar^2}.$$

General solution:

$$\psi = A \sin(kx) + B \cos(kx).$$

Boundary at $x = 0$:

$$\psi(0) = B = 0.$$

So

$$\psi = A \sin(kx).$$

Boundary at $x = a$:

$$A \sin(ka) = 0.$$

Nonzero A requires

$$ka = n\pi, \quad n = 1, 2, 3, \dots$$

so

$$k_n = \frac{n\pi}{a}.$$

Energy:

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2ma^2} = \frac{n^2 h^2}{8ma^2}.$$

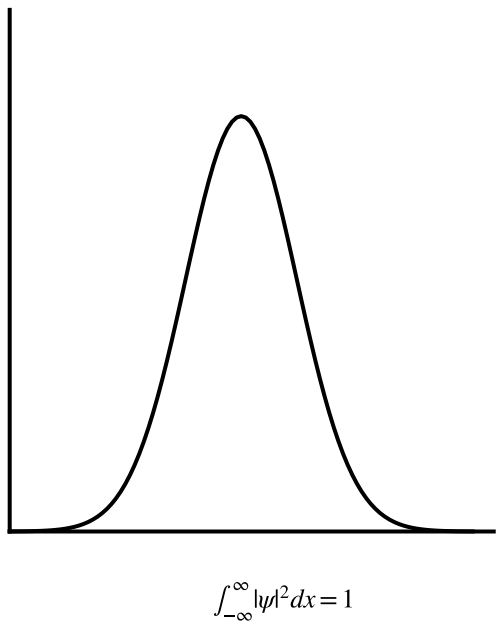
Normalize:

$$1 = A^2 \int_0^a \sin^2 \frac{n\pi x}{a} dx = A^2 \frac{a}{2}.$$

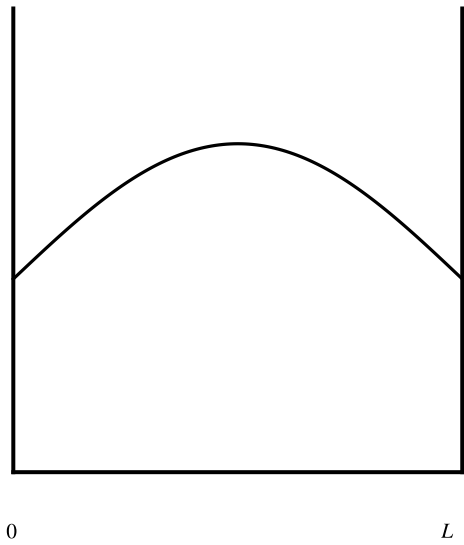
Thus

$$A = \sqrt{\frac{2}{a}}.$$

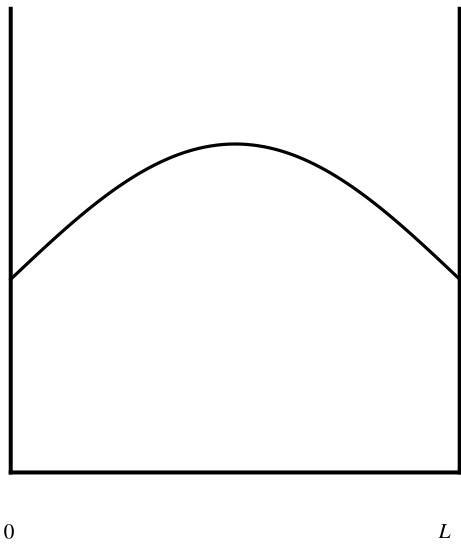
Q29. Normalize a Gaussian wavefunction



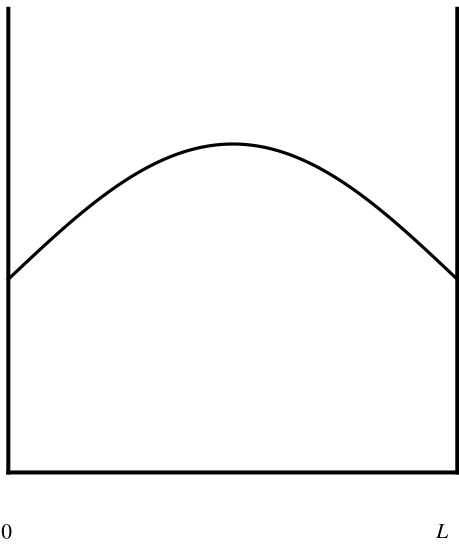
Q30. Expectation value $\langle x \rangle$ in an infinite box



Q31. Verify a time-dependent box solution



Q32. Solve the infinite one-dimensional well



Final answer

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}, \quad E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}.$$

Question 33 - Infinite box centred at the origin

Source: QOE Quantum Assignment 2025 Q1

Complete question. A particle of mass m is in a one-dimensional box from $x = -L/2$ to $x = L/2$. Determine the normalized energy eigenfunctions and eigenvalues.

Inside the box:

$$\psi = A \sin(kx) + B \cos(kx), \quad E = \frac{\hbar^2 k^2}{2m}.$$

Boundary conditions:

$$\psi(-L/2) = 0, \quad \psi(L/2) = 0.$$

Because the box is symmetric, solutions have definite parity.

Even parity: $\psi = B \cos(kx)$.

$$\cos(kL/2) = 0 \quad \Rightarrow \quad kL/2 = \frac{(2q+1)\pi}{2}.$$

Thus

$$k = \frac{(2q+1)\pi}{L},$$

which corresponds to odd n .

Odd parity: $\psi = A \sin(kx)$.

$$\sin(kL/2) = 0 \quad \Rightarrow \quad kL/2 = q\pi.$$

Thus

$$k = \frac{2q\pi}{L},$$

which corresponds to even n .

Normalization gives the same constant $\sqrt{2/L}$.

Final answer

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right), & n = 1, 3, 5, \dots \\ \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), & n = 2, 4, 6, \dots \end{cases}$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}.$$

Question 34 - Eigenfunctions of D and D^2

Source: QOE Quantum Assignment 2025 Q2

Complete question. Determine eigenfunctions of

$$D = \frac{d}{dx}, \quad D^2 = \frac{d^2}{dx^2}.$$

For D , require

$$Df = \lambda f.$$

That is

$$\frac{df}{dx} = \lambda f.$$

Separate variables:

$$\frac{df}{f} = \lambda dx.$$

Integrate:

$$\ln f = \lambda x + C.$$

So

$$f(x) = Ce^{\lambda x}.$$

Hence exponentials are eigenfunctions of D with eigenvalue λ .

For D^2 :

$$D^2 f = \mu f.$$

Examples:

$$\begin{aligned} D^2 e^{kx} &= k^2 e^{kx}, & D^2 e^{-kx} &= k^2 e^{-kx}, \\ D^2 \sin(kx) &= -k^2 \sin(kx), & D^2 \cos(kx) &= -k^2 \cos(kx). \end{aligned}$$

Final answer

$e^{\lambda x}$ is an eigenfunction of D . Exponentials, sine and cosine families are eigenfunctions of D^2 with the eigenvalues shown above.

Question 35 - Hermitian adjoint of a complex and a real scalar

Source: QOE Quantum Assignment 2025 Q3

Complete question. Prove from the definition of adjoint that (a) if $\hat{A} = c$ where c is complex, then $\hat{A}^\dagger = c^*$; (b) if $\hat{A} = a$ where a is real, then $\hat{A}^\dagger = a$.

Definition:

$$\langle \psi | \hat{A} \phi \rangle = \langle \hat{A}^\dagger \psi | \phi \rangle.$$

For $\hat{A} = c$:

$$\langle \psi | c \phi \rangle = c \int \psi^* \phi \, dx.$$

We want to write the integrand as $(\hat{A}^\dagger \psi)^* \phi$. Since

$$(c^* \psi)^* = c \psi^*,$$

we have

$$c \int \psi^* \phi \, dx = \int (c^* \psi)^* \phi \, dx.$$

Therefore

$$\hat{A}^\dagger = c^*.$$

If a is real, then $a^* = a$, so

Final answer

$$\boxed{c^\dagger = c^*}, \quad \boxed{a^\dagger = a \text{ for real } a}.$$

Question 36 - Hermitian adjoint of D^2

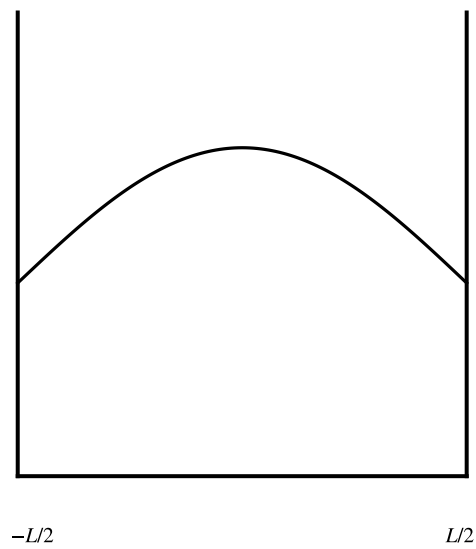
Source: QOE Quantum Assignment 2025 Q4

Complete question. Determine the Hermitian adjoint of

$$D^2 = \frac{d^2}{dx^2},$$

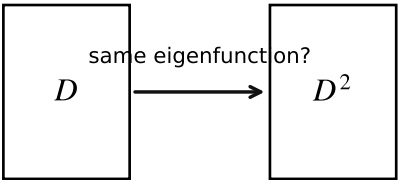
assuming physical wavefunctions vanish at the boundaries.

Q33. Infinite box centred at the origin



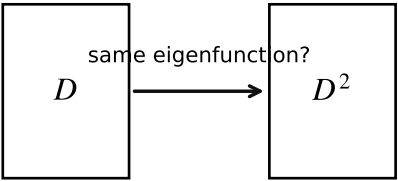
Q34. Eigenfunctions of D and D^2

$$e^{ax}, \sin kx, \cos kx$$



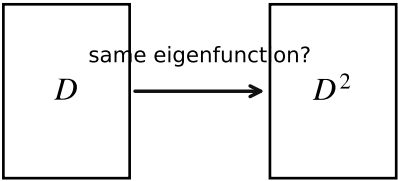
Q35. Hermitian adjoint of a complex and a real scalar

$$c^\dagger = c^*, \quad a^\dagger = a \ (a \in \mathbb{R})$$



Q36. Hermitian adjoint of D^2

$$(D^2)^\dagger = D^2$$



Start from

$$\int \psi^* \phi'' dx.$$

Integrate by parts once:

$$\int \psi^* \phi'' dx = [\psi^* \phi'] - \int \psi'^* \phi' dx.$$

Boundary term vanishes. Integrate the remaining term by parts:

$$- \int \psi'^* \phi' dx = -[\psi'^* \phi] + \int \psi''^* \phi dx.$$

The second boundary term also vanishes. Therefore

$$\int \psi^* D^2 \phi dx = \int (D^2 \psi)^* \phi dx.$$

By the adjoint definition,

Final answer

$$(D^2)^\dagger = D^2.$$

So the second-derivative operator is Hermitian under the stated boundary conditions.

Question 37 - Same eigenfunctions imply commuting operators

Source: QOE Quantum Assignment 2025 Q5

Complete question. If $\hat{A}$ and $\hat{B}$ have the same complete set of eigenfunctions, show that they commute.

Let

$$\hat{A}\phi_i = a_i\phi_i, \quad \hat{B}\phi_i = b_i\phi_i.$$

Act with $\hat{A}\hat{B}$:

$$\hat{A}\hat{B}\phi_i = \hat{A}(b_i\phi_i) = b_i\hat{A}\phi_i = a_ib_i\phi_i.$$

Reverse order:

$$\hat{B}\hat{A}\phi_i = \hat{B}(a_i\phi_i) = a_i\hat{B}\phi_i = a_ib_i\phi_i.$$

Thus

$$(\hat{A}\hat{B} - \hat{B}\hat{A})\phi_i = 0.$$

Because the ϕ_i form a complete basis, this holds for any state.

Final answer

$$[\hat{A}, \hat{B}] = 0.$$

Question 38 - Ground-state box: $\langle x \rangle$ and probability in $0 < x < L/4$

Source: Physics 5 Test 2 2023 Q2

Complete question. For

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L},$$

find (a) $\langle x \rangle$ and (b) $P(0 < x < L/4)$.

Part (a) is the general result of Question 30:

$$\langle x \rangle = L/2.$$

For part (b):

$$P = \frac{2}{L} \int_0^{L/4} \sin^2 \frac{\pi x}{L} dx.$$

Use $\sin^2 u = (1 - \cos 2u)/2$:

$$P = \frac{1}{L} \int_0^{L/4} \left(1 - \cos \frac{2\pi x}{L}\right) dx.$$

Integrate:

$$P = \frac{1}{L} \left[x - \frac{L}{2\pi} \sin \frac{2\pi x}{L} \right]_0^{L/4}.$$

At $x = L/4$, $\sin(\pi/2) = 1$:

$$P = \frac{1}{4} - \frac{1}{2\pi} = 0.0908.$$

Final answer

$$\langle x \rangle = L/2, \quad P(0 < x < L/4) = 0.0908.$$

Question 39 - First excited box state: most probable positions and probability

Source: Physics 5 Test 2 2022 Q3

Complete question. For the first excited state $n = 2$ of a box $0 < x < L$, find (a) the most probable positions and (b) $P(0 < x < L/4)$.

$$|\psi_2|^2 = \frac{2}{L} \sin^2 \frac{2\pi x}{L}.$$

(a) Most probable positions Probability density is maximum when

$$\sin^2 \frac{2\pi x}{L} = 1.$$

So

$$\frac{2\pi x}{L} = \frac{\pi}{2}, \frac{3\pi}{2}.$$

Hence

$$x = L/4, \quad x = 3L/4.$$

(b) Probability $0 < x < L/4$

$$P = \frac{2}{L} \int_0^{L/4} \sin^2 \frac{2\pi x}{L} dx.$$

$$P = \frac{1}{L} \left[x - \frac{L}{4\pi} \sin \frac{4\pi x}{L} \right]_0^{L/4}.$$

Since $\sin \pi = 0$,

$$P = \frac{1}{L} \left(\frac{L}{4} \right) = 0.250.$$

Final answer

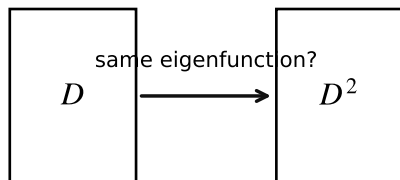
$$x = L/4, 3L/4, \quad P = 0.250.$$

Question 40 - $n = 2$ box: $\langle x \rangle$ and probability near the centre

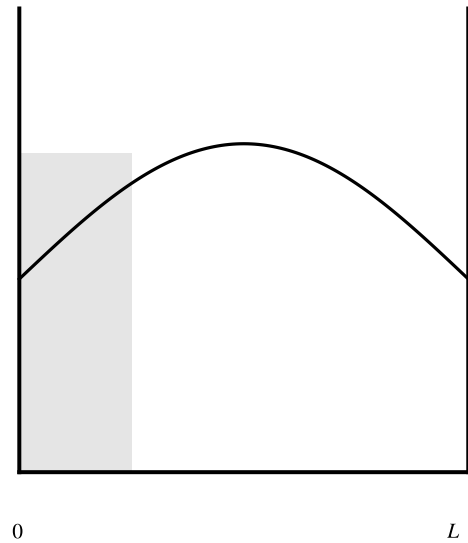
Source: Physics 5 Test 2 2024 Q5

Q37. Same eigenfunctions imply commuting operators

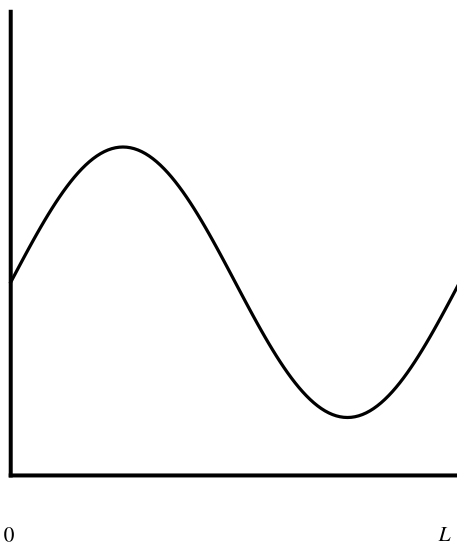
$[\hat{A}, \hat{B}] = 0$ for a common complete eigenbasis



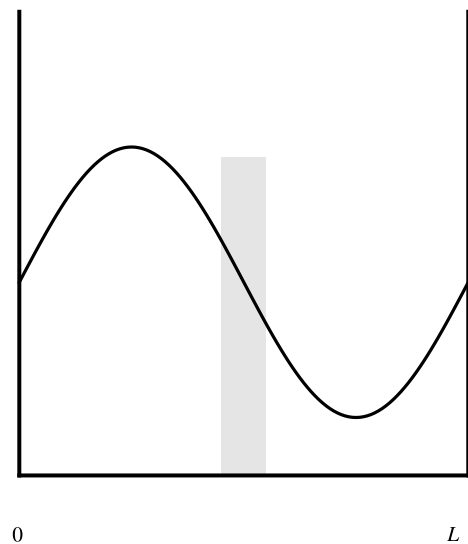
Q38. Ground-state box: $\langle x \rangle$ and probability in $0 < x < L/4$



Q39. First excited box state: most probable positions



Q40. $n=2$ box: $\langle x \rangle$ and probability near the centre



Complete question. For

$$\psi_2(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L},$$

find (a) $\langle x \rangle$ and (b) the probability for $0.490L \leq x \leq 0.510L$.

By symmetry,

$$\langle x \rangle = L/2.$$

Probability:

$$P = \frac{2}{L} \int_{0.490L}^{0.510L} \sin^2 \frac{2\pi x}{L} dx.$$

Using $\sin^2 u = (1 - \cos 2u)/2$:

$$P = \frac{1}{L} \left[x - \frac{L}{4\pi} \sin \frac{4\pi x}{L} \right]_{0.490L}^{0.510L}.$$

Let $u = x/L$:

$$P = 0.020 - \frac{1}{4\pi} [\sin(2.04\pi) - \sin(1.96\pi)].$$

Since

$$\sin(2.04\pi) = \sin(0.04\pi), \quad \sin(1.96\pi) = -\sin(0.04\pi),$$

$$P = 0.020 - \frac{2 \sin(0.04\pi)}{4\pi} \approx 5.26 \times 10^{-5}.$$

Final answer

$$\langle x \rangle = L/2, \quad P = 5.26 \times 10^{-5}.$$

Question 41 - Photon from a 5 nm well, $n = 5 \rightarrow 2$

Source: Physics 5 Test 2 2024 Q4

Complete question. An electron in a one-dimensional infinite well of length $L = 5$ nm drops from level 5 to level 2. Find the emitted wavelength.

$$E_n = \frac{n^2 h^2}{8mL^2}.$$

Energy difference:

$$\Delta E = E_5 - E_2 = (25 - 4) \frac{h^2}{8mL^2} = 21 \frac{h^2}{8mL^2}.$$

Substitute:

$$\Delta E = 21 \frac{(6.626 \times 10^{-34})^2}{8(9.11 \times 10^{-31})(5.00 \times 10^{-9})^2} = 5.06 \times 10^{-20} \text{ J}.$$

Then

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{5.06 \times 10^{-20}} = 3.93 \times 10^{-6} \text{ m}.$$

Final answer

$$\lambda = 3.93 \text{ } \mu\text{m}$$

Question 42 - Solve the two-dimensional infinite box

Source: Physics 5 Test 2 2022 Q4

Complete question. A particle of mass m is in a 2D box $0 < x < L_x$, $0 < y < L_y$ with infinite walls. Use separation of variables to obtain the normalized wavefunction and energies.

Inside:

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) = E\psi.$$

Assume

$$\psi(x, y) = X(x)Y(y).$$

Substitute:

$$-\frac{\hbar^2}{2m} (YX'' + XY'') = EXY.$$

Divide by XY :

$$\frac{X''}{X} + \frac{Y''}{Y} = -\frac{2mE}{\hbar^2}.$$

Each term must be a constant:

$$X'' + k_x^2 X = 0, \quad Y'' + k_y^2 Y = 0.$$

Boundary conditions give

$$k_x = \frac{n_x \pi}{L_x}, \quad k_y = \frac{n_y \pi}{L_y}.$$

Normalized factors:

$$X = \sqrt{\frac{2}{L_x}} \sin \frac{n_x \pi x}{L_x}, \quad Y = \sqrt{\frac{2}{L_y}} \sin \frac{n_y \pi y}{L_y}.$$

Therefore

Final answer

$$\psi_{n_x n_y} = \frac{2}{\sqrt{L_x L_y}} \sin \frac{n_x \pi x}{L_x} \sin \frac{n_y \pi y}{L_y}$$

$$E_{n_x n_y} = \frac{\pi^2 \hbar^2}{2m} \left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right).$$

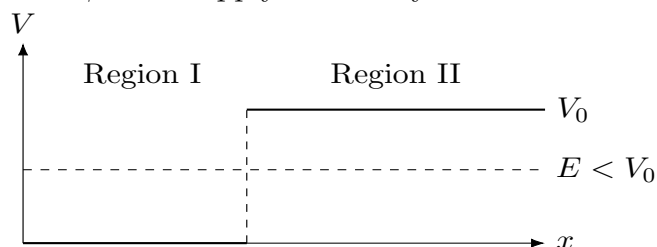
Question 43 - Potential step for $E < V_0$

Source: Physics 5 Test 2 2023 Q3

Complete question. Particles come from $x = -\infty$ toward

$$V = 0 \ (x < 0), \quad V = V_0 \ (x \geq 0),$$

with $E < V_0$. Determine ψ_I and ψ_{II} and apply continuity at $x = 0$.



Region I:

$$\psi_I = Ae^{ikx} + Be^{-ikx}, \quad k = \frac{\sqrt{2mE}}{\hbar}.$$

Region II:

$$\frac{d^2 \psi}{dx^2} = \kappa^2 \psi, \quad \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}.$$

General solution is $Ce^{-\kappa x} + De^{\kappa x}$. Since $e^{\kappa x}$ diverges as $x \rightarrow \infty$, set $D = 0$:

$$\psi_{II} = Ce^{-\kappa x}.$$

Continuity of ψ at $x = 0$:

$$A + B = C.$$

Continuity of derivative:

$$ik(A - B) = -\kappa C.$$

Use $C = A + B$:

$$ik(A - B) = -\kappa(A + B).$$

Collect B terms:

$$(ik + \kappa)A = (ik - \kappa)B.$$

Thus

$$\frac{B}{A} = \frac{ik + \kappa}{ik - \kappa} = \frac{k - i\kappa}{k + i\kappa}.$$

Then

$$C = A + B = \frac{2k}{k + i\kappa}A = \frac{2ik}{ik - \kappa}A.$$

Final answer

$$\psi_I = Ae^{ikx} + Be^{-ikx}, \quad \psi_{II} = Ce^{-\kappa x}.$$

The wave penetrates into the classically forbidden region but decays exponentially.

Question 44 - Harmonic oscillator ground state

Source: QOE Test 2 2024 Q4

Complete question. For

$$V(x) = \frac{1}{2}m\omega_0^2 x^2, \quad \psi = Ae^{-x^2/(2a^2)},$$

substitute into the Schrodinger equation and find a and E .

First derivative:

$$\frac{d\psi}{dx} = -\frac{x}{a^2}\psi.$$

Second derivative:

$$\frac{d^2\psi}{dx^2} = \frac{d}{dx} \left(-\frac{x}{a^2}\psi \right) = -\frac{1}{a^2}\psi - \frac{x}{a^2} \frac{d\psi}{dx}.$$

Substitute $d\psi/dx = -(x/a^2)\psi$:

$$\frac{d^2\psi}{dx^2} = \left(-\frac{1}{a^2} + \frac{x^2}{a^4} \right) \psi.$$

Schrodinger equation:

$$-\frac{\hbar^2}{2m} \left(-\frac{1}{a^2} + \frac{x^2}{a^4} \right) \psi + \frac{1}{2}m\omega_0^2 x^2 \psi = E\psi.$$

Divide by ψ :

$$\frac{\hbar^2}{2ma^2} + \left[-\frac{\hbar^2}{2ma^4} + \frac{1}{2}m\omega_0^2 \right] x^2 = E.$$

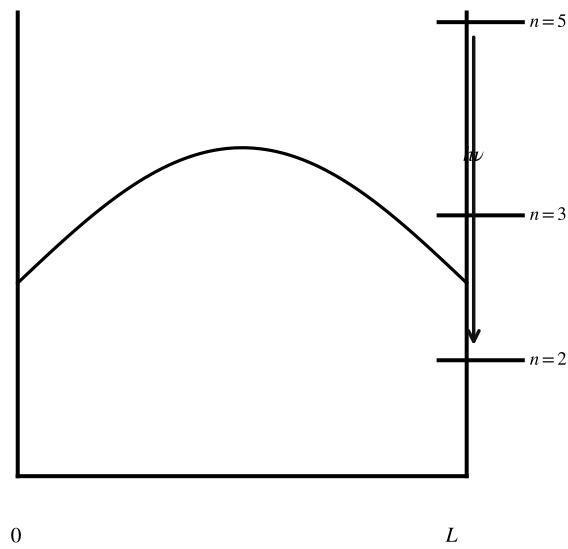
Because E is constant, coefficient of x^2 must be zero:

$$-\frac{\hbar^2}{2ma^4} + \frac{1}{2}m\omega_0^2 = 0.$$

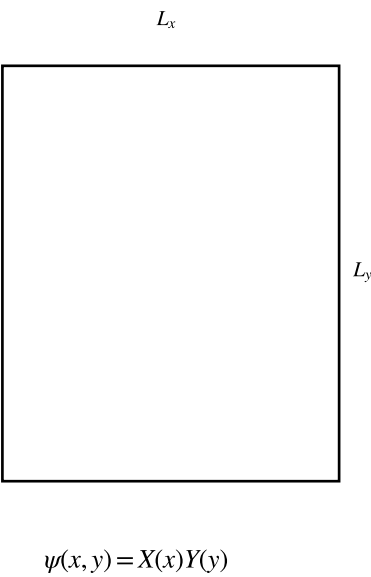
Multiply by 2:

$$\frac{\hbar^2}{ma^4} = m\omega_0^2.$$

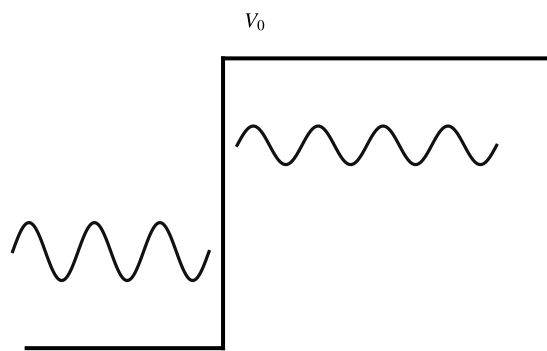
Q41. Photon from a 5 nm well, $n=5 \rightarrow 2$



Q42. Solve the two-dimensional infinite box

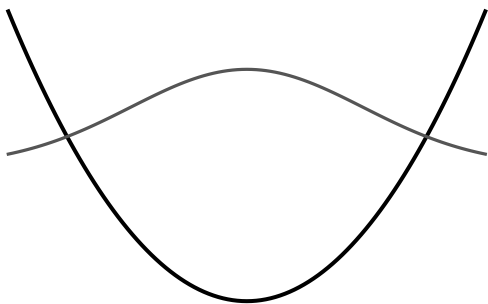


Q43. Potential step for $E < V_0$



Q44. Harmonic oscillator ground state

$$V(x) = \frac{1}{2}m\omega^2x^2$$



$$a^4 = \frac{\hbar^2}{m^2 \omega_0^2}.$$

$$a^2 = \frac{\hbar}{m \omega_0}, \quad a = \sqrt{\frac{\hbar}{m \omega_0}}.$$

Constant term:

$$E = \frac{\hbar^2}{2ma^2} = \frac{\hbar^2}{2m} \left(\frac{m \omega_0}{\hbar} \right) = \frac{1}{2} \hbar \omega_0.$$

Final answer

$$a = \sqrt{\frac{\hbar}{m \omega_0}}, \quad E_0 = \frac{1}{2} \hbar \omega_0.$$

Question 45 - Harmonic oscillator first excited state

Source: Physics 5 Test 2 2022 Q5

Complete question. For

$$V(x) = \frac{1}{2} m \omega_0^2 x^2, \quad \psi = A x e^{-ax^2},$$

(a) determine a and E by substitution; (b) normalize to find A .

Let $f = x e^{-ax^2}$, so $\psi = A f$.

First derivative:

$$f' = e^{-ax^2} - 2ax^2 e^{-ax^2} = e^{-ax^2} (1 - 2ax^2).$$

Second derivative:

$$\begin{aligned} f'' &= (-2ax) e^{-ax^2} (1 - 2ax^2) + e^{-ax^2} (-4ax) \\ &= e^{-ax^2} [-2ax + 4a^2 x^3 - 4ax] \\ &= e^{-ax^2} (-6ax + 4a^2 x^3) \\ &= (-6a + 4a^2 x^2) f. \end{aligned}$$

Thus

$$\psi'' = (-6a + 4a^2 x^2) \psi.$$

Substitute:

$$-\frac{\hbar^2}{2m} (-6a + 4a^2 x^2) \psi + \frac{1}{2} m \omega_0^2 x^2 \psi = E \psi.$$

Divide by ψ :

$$\frac{3\hbar^2 a}{m} + \left(-\frac{2\hbar^2 a^2}{m} + \frac{1}{2} m \omega_0^2 \right) x^2 = E.$$

Set x^2 coefficient to zero:

$$\frac{2\hbar^2 a^2}{m} = \frac{1}{2} m \omega_0^2.$$

$$4\hbar^2 a^2 = m^2 \omega_0^2.$$

$$a = \frac{m \omega_0}{2\hbar}.$$

Energy:

$$E = \frac{3\hbar^2 a}{m} = \frac{3\hbar^2}{m} \frac{m \omega_0}{2\hbar} = \frac{3}{2} \hbar \omega_0.$$

Normalization:

$$1 = A^2 \int_{-\infty}^{\infty} x^2 e^{-2ax^2} dx.$$

Use

$$\int_{-\infty}^{\infty} x^2 e^{-\beta x^2} dx = \frac{\sqrt{\pi}}{2\beta^{3/2}}.$$

With $\beta = 2a$:

$$1 = A^2 \frac{\sqrt{\pi}}{2(2a)^{3/2}}.$$

Thus

$$A^2 = \frac{2(2a)^{3/2}}{\sqrt{\pi}}.$$

$$A = \frac{2^{5/4} a^{3/4}}{\pi^{1/4}}.$$

Substitute $a = m\omega_0/(2\hbar)$:

Final answer

$$a = \frac{m\omega_0}{2\hbar}, \quad E = \frac{3}{2}\hbar\omega_0, \quad A = \left(\frac{4}{\pi}\right)^{1/4} \left(\frac{m\omega_0}{\hbar}\right)^{3/4}.$$

Question 46 - Hydrogen ground-state average radius

Source: QOE Test 2 2024 Q5

Complete question. The hydrogen ground-state wavefunction is

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}.$$

Determine $\langle r \rangle$ from first principles.

$$\langle r \rangle = \int \psi^* r \psi dV, \quad dV = r^2 \sin \theta dr d\theta d\phi.$$

Thus

$$\langle r \rangle = \frac{1}{\pi a_0^3} \int_0^\infty r^3 e^{-2r/a_0} dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi.$$

Angular integrals:

$$\int_0^\pi \sin \theta d\theta = 2, \quad \int_0^{2\pi} d\phi = 2\pi.$$

Therefore

$$\langle r \rangle = \frac{4}{a_0^3} \int_0^\infty r^3 e^{-2r/a_0} dr.$$

Let

$$x = \frac{2r}{a_0}, \quad r = \frac{a_0 x}{2}, \quad dr = \frac{a_0}{2} dx.$$

Then

$$\langle r \rangle = \frac{4}{a_0^3} \int_0^\infty \left(\frac{a_0 x}{2}\right)^3 e^{-x} \left(\frac{a_0}{2}\right) dx.$$

$$\langle r \rangle = \frac{4a_0^4}{16a_0^3} \int_0^\infty x^3 e^{-x} dx.$$

Use $\int_0^\infty x^n e^{-x} dx = n!$:

$$\langle r \rangle = \frac{a_0}{4} (3!) = \frac{a_0}{4} (6) = \frac{3}{2} a_0.$$

Final answer

$$\langle r \rangle = \frac{3}{2} a_0$$

Question 47 - Most likely position in the hydrogen $2p$, $m_l = 0$ state

Source: QOE Test 2 2024 Q6

Complete question. For $n = 2$, $l = 1$, $m_l = 0$, determine the most likely position of the electron. After including the spherical volume factor and integrating over angles, the radial probability is

$$P(r) = \frac{1}{24a_0^5} r^4 e^{-r/a_0}.$$

The maximum occurs when $dP/dr = 0$. Ignore the positive constant and differentiate:

$$\frac{d}{dr}(r^4 e^{-r/a_0}) = 4r^3 e^{-r/a_0} - \frac{r^4}{a_0} e^{-r/a_0}.$$

Factor:

$$r^3 e^{-r/a_0} \left(4 - \frac{r}{a_0} \right) = 0.$$

Nonzero maximum:

$$4 - \frac{r}{a_0} = 0 \quad \Rightarrow \quad r = 4a_0.$$

Numerically:

$$r = 4(5.29 \times 10^{-11}) = 2.12 \times 10^{-10} \text{ m}.$$

For this state, angular dependence is $|\psi|^2 \propto \cos^2 \theta$, which is maximum at $\theta = 0$ and π .

Final answer

$$r_{\text{mp}} = 4a_0 = 2.12 \times 10^{-10} \text{ m},$$

most likely along the $+z$ or $-z$ axis.

Question 48 - Free-electron wavefunction

Source: QM tutorial / Serway P40.1

Complete question. A free electron has $\psi = Ae^{i(5.00 \times 10^{10})x}$. Find (a) de Broglie wavelength, (b) momentum and (c) kinetic energy in eV.

$$k = 5.00 \times 10^{10} \text{ m}^{-1}.$$

(a)

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{5.00 \times 10^{10}} = 1.26 \times 10^{-10} \text{ m} = 0.126 \text{ nm}.$$

(b)

$$p = \hbar k = (1.055 \times 10^{-34})(5.00 \times 10^{10}) = 5.27 \times 10^{-24} \text{ kg m/s}.$$

(c)

$$K = \frac{p^2}{2m_e} = \frac{(5.27 \times 10^{-24})^2}{2(9.11 \times 10^{-31})} = 1.53 \times 10^{-17} \text{ J}.$$

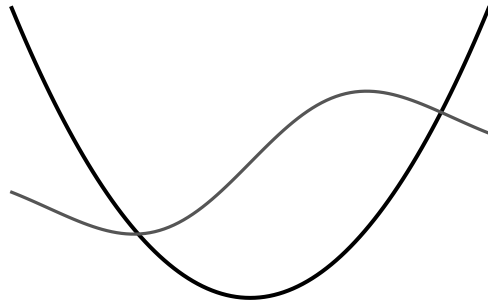
$$K = \frac{1.53 \times 10^{-17}}{1.602 \times 10^{-19}} = 95.4 \text{ eV}.$$

Final answer

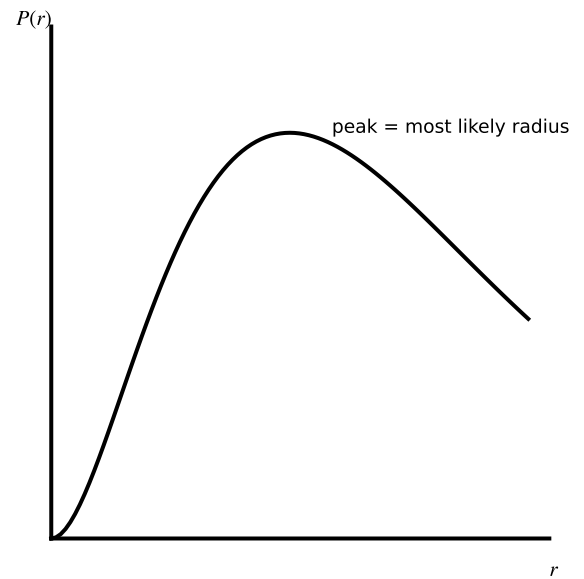
$$\lambda = 0.126 \text{ nm}, \quad p = 5.27 \times 10^{-24} \text{ kg m/s}, \quad K = 95.4 \text{ eV}.$$

Q45. Harmonic oscillator first excited state

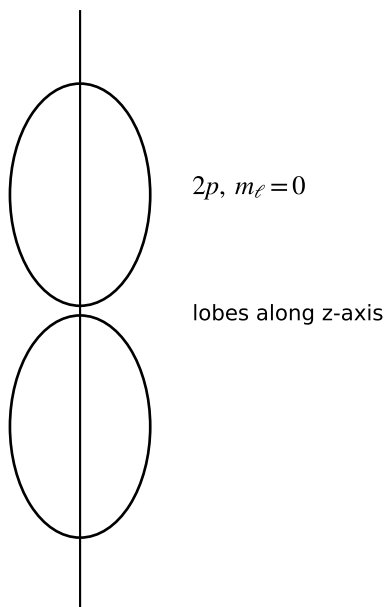
$$V(x) = \frac{1}{2}m\omega^2 x^2$$



Q46. Hydrogen ground-state average radius

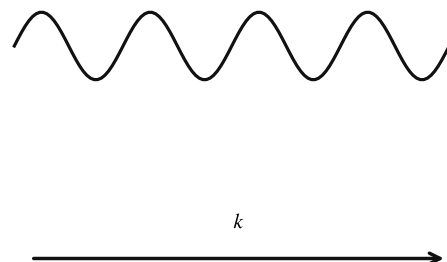


Q47. Most likely position in the hydrogen $2p, m_\ell = 0$ state



Q48. Free-electron wavefunction

$$\psi \propto e^{i(kx - \omega t)}$$



Question 49 - Tunnelling through a 0.100 nm barrier

Source: QM tutorial / Serway P40.33

Complete question. Estimate the transmission probability through a barrier of width $L = 0.100$ nm for an electron with energy deficit (a) $U - E = 0.0100$ eV and (b) 1.00 eV.

Use

$$T \approx e^{-2\kappa L}, \quad \kappa = \frac{\sqrt{2m_e(U - E)}}{\hbar}.$$

(a) Convert energy deficit:

$$U - E = 0.0100(1.602 \times 10^{-19}) = 1.602 \times 10^{-21} \text{ J}.$$

$$\kappa = \frac{\sqrt{2(9.11 \times 10^{-31})(1.602 \times 10^{-21})}}{1.055 \times 10^{-34}} = 5.12 \times 10^8 \text{ m}^{-1}.$$

$$T = e^{-2(5.12 \times 10^8)(1.00 \times 10^{-10})} = e^{-0.1024} = 0.903.$$

(b) 1.00 eV is 100 times the energy deficit, so κ is 10 times larger:

$$\kappa = 5.12 \times 10^9 \text{ m}^{-1}.$$

$$T = e^{-2(5.12 \times 10^9)(1.00 \times 10^{-10})} = e^{-1.024} = 0.359.$$

Final answer

$$T \approx 0.903 \text{ for } 0.0100 \text{ eV}, \quad T \approx 0.359 \text{ for } 1.00 \text{ eV}.$$

Question 50 - Uncertainty estimate for an electron confined to 0.1 nm

Source: Chapter 7 worked example

Complete question. Estimate the minimum momentum uncertainty and corresponding kinetic-energy scale for an electron confined to $\Delta x = 0.1$ nm.

$$\Delta x \Delta p \geq \frac{\hbar}{2}.$$

$$\Delta p \geq \frac{1.055 \times 10^{-34}}{2(0.1 \times 10^{-9})} = 5.28 \times 10^{-25} \text{ kg m/s}.$$

Estimate

$$K \sim \frac{(\Delta p)^2}{2m_e} = \frac{(5.28 \times 10^{-25})^2}{2(9.11 \times 10^{-31})} = 1.53 \times 10^{-19} \text{ J}.$$

Convert to eV:

$$K \approx \frac{1.53 \times 10^{-19}}{1.602 \times 10^{-19}} = 0.95 \text{ eV}.$$

Final answer

$$\Delta p \gtrsim 5.28 \times 10^{-25} \text{ kg m/s}, \quad K \sim 0.95 \text{ eV}.$$

Question 51 - Lowest energy in a 0.200 nm box

Source: QM tutorial / Serway P40.6

Complete question. Find the lowest possible energy of (a) a proton and (b) an electron in a one-dimensional box of width 0.200 nm.

$$E_1 = \frac{h^2}{8mL^2}, \quad L = 0.200 \times 10^{-9} \text{ m}.$$

Proton:

$$E_{1p} = \frac{(6.626 \times 10^{-34})^2}{8(1.67 \times 10^{-27})(0.200 \times 10^{-9})^2} = 8.21 \times 10^{-22} \text{ J.}$$

$$E_{1p} = 5.13 \times 10^{-3} \text{ eV.}$$

Electron:

$$E_{1e} = \frac{(6.626 \times 10^{-34})^2}{8(9.11 \times 10^{-31})(0.200 \times 10^{-9})^2} \approx 1.51 \times 10^{-18} \text{ J.}$$

$$E_{1e} \approx 9.4 \text{ eV.}$$

Final answer

$$E_{1p} \approx 5.13 \times 10^{-3} \text{ eV}, \quad E_{1e} \approx 9.4 \text{ eV}.$$

The electron energy is much larger because $E \propto 1/m$.

Final rapid memory sheet**Laser**

$$N_1 B \rho = N_2 A + N_2 B \rho$$

$$N_2/N_1 = e^{-\Delta E/k_B T}$$

$$A = 1/\tau$$

$$\alpha = \lambda^2(N_2 - N_1)A/(8\pi\Delta\nu)$$

$$\Delta\nu \simeq c\Delta\lambda/\lambda^2$$

$$\alpha_{th} = \gamma - (1/2L) \ln(R_1 R_2)$$

$$R = [(n-1)/(n+1)]^2$$

$$\Delta\nu_D \simeq 2v\nu_0/c$$

Quantum

$$E = hf = hc/\lambda$$

$$K_{max} = hf - \phi$$

$$\lambda' = \lambda + \lambda_C(1 - \cos \theta)$$

$$\lambda_{dB} = h/p$$

$$E_n(H) = -13.6/n^2 \text{ eV}$$

$$-\hbar^2 \nabla^2 \psi / (2m) + V\psi = E\psi$$

$$\psi_n = \sqrt{2/L} \sin(n\pi x/L)$$

$$E_n = n^2 \hbar^2 / (8mL^2)$$

$$E_n^{HO} = (n + 1/2)\hbar\omega$$

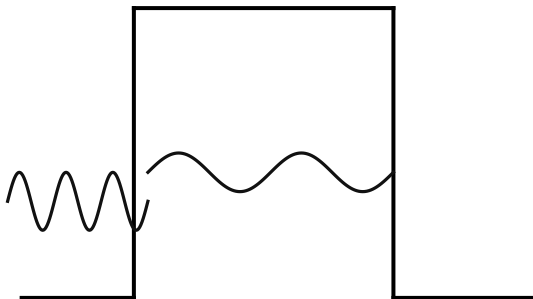
$$\langle r \rangle_{1s} = 3a_0/2$$

Source coverage index - Test 2 only

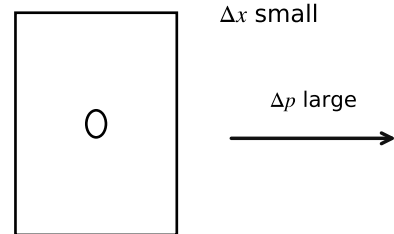
This index shows where the Test 2-relevant questions from the supplied documents are covered. Repeated questions are intentionally merged.

Source document	Relevant question(s)	Covered here
Tutorial 4 - Laser Theory	Q1-Q6	Questions 2, 4-8
Memo Tutorial 4	Worked solutions Q1-Q6	Questions 2, 4-8
Laser Theory Chapter 5	Ex. 5.1, Ex. 5.2, Example 5.1, Einstein processes/gain	Questions 1-3, 13-14
GA Assignment 1 Laser Theory	Q1-Q16	Questions 1, 13 and rapid laser drill
QOE Test 2 2023	Q4-Q7 (laser only)	Questions 1-3, 9-10
QOE Test 2 2024	Q2-Q6 (excluding Chapter 4 Q1)	Questions 11-12, 44, 46-47
QM Practice Questions 2025	Sections A-D and bonus	Questions 15, 27 and quantum rapid drill
Failure of Classical Mechanics Qs	Conceptual set	Quantum rapid drill

Q49. Tunnelling through a 0.100 nm barrier

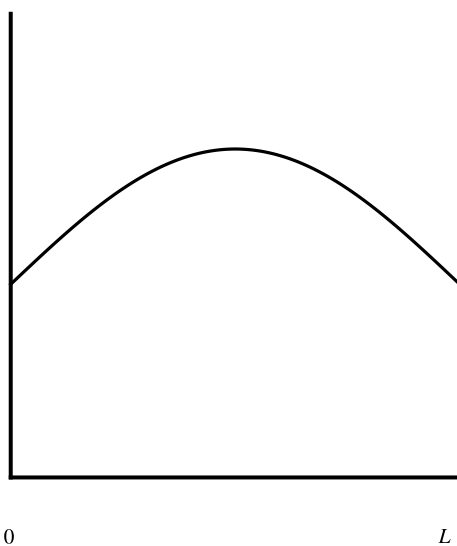


Q50. Uncertainty estimate for an electron confined to 0.1 nm



$$\Delta x \Delta p \geq \hbar/2$$

Q51. Lowest energy in a 0.200 nm box



Source document	Relevant question(s)	Covered here
Assignment 1 Atomic and Quantum Physics	Q1-Q5	Questions 17-21 plus Sun/Wien material
Physics 5 Test 2 2024	Q1-Q5 quantum	Questions 16, 19, 23, 41, 40
Physics 5 Test 2 2023	Q1-Q3 quantum	Questions 22, 38, 43
Physics 5 Test 2 2022	Q3-Q5 quantum	Questions 39, 42, 45
Some Solved Problems	Q1-Q4	Questions 29-32
QOE Quantum Assignment 2025	Q1-Q5	Questions 33-37
QM Tutorial / selected Chapter 40 problems identified in Test 2 pack	Free electron, box energy, tunnelling, uncertainty	Questions 48-51

Exam note

Not included because they are outside the confirmed current Test 2 scope: Tutorial 1 and memo, Tutorial 2 vectorial light/polarization and memo, Tutorial 3 reflection/refraction/fibres and memo, Fabry-Perot Chapter 4 questions, Brewster-angle Chapter 4 questions, and magnetostatics questions from older Physics 5 tests.

ADDED TEST 2 MATERIAL

Selected only from the newly attached Physics 5 documentation packs

Included because it is directly useful for Test 2

- Math tools used in quantum calculations: integration by parts, Gaussian integrals and trig identities.
- Black-body radiation: Stefan-Boltzmann law and Wien law, with worked example.
- Photoelectric effect: photon energy, work function and maximum kinetic energy.
- Compton scattering: wavelength shift, photon energy and transferred kinetic energy.
- Particle in a box: infinite well, symmetric well, normalization, expectation values and transitions.
- Hydrogen ground-state radial probability and orbital angular momentum.

Not included from the attached packs

- Electrostatics and Gauss-law chapters
- Magnetostatics / Biot-Savart / Ampere-law chapters
- Waves and optics topics outside the confirmed Test 2 scope

1 Calculus and Trigonometry Tools

1.1 Differentiation rules used in Physics 5

Concept

$$\begin{aligned}\frac{d}{dx}(x^n) &= nx^{n-1}, \\ \frac{d}{dx}(x^{-2}) &= -2x^{-3} = -\frac{2}{x^3}, \\ \frac{d}{dx}(x^{-1}) &= -x^{-2} = -\frac{1}{x^2}.\end{aligned}$$

For a composite reciprocal function:

$$\begin{aligned}\frac{d}{dz} [(z^2 + 1)^{-1}] &= -(z^2 + 1)^{-2}(2z) \\ &= -\frac{2z}{(z^2 + 1)^2}.\end{aligned}$$

1.2 Integration by parts

Concept

$$\int u \, dv = uv - \int v \, du.$$

Common Physics 5 use:

$$\int x \cos(kx) \, dx, \quad \int x \sin(kx) \, dx, \quad \int x^2 \ln x \, dx.$$

Example: integral needed for expectation value

Evaluate

$$\int x \cos(kx) \, dx.$$

Choose

$$u = x, \quad dv = \cos(kx) \, dx.$$

Then

$$du = dx, \quad v = \frac{\sin(kx)}{k}.$$

Therefore

$$\begin{aligned}\int x \cos(kx) \, dx &= \frac{x \sin(kx)}{k} - \int \frac{\sin(kx)}{k} \, dx \\ &= \frac{x \sin(kx)}{k} + \frac{\cos(kx)}{k^2} + C.\end{aligned}$$

1.3 Gaussian integral and normalization

Concept

$$\int_{-\infty}^{\infty} e^{-bx^2} \, dx = \sqrt{\frac{\pi}{b}}.$$

If

$$\psi(x) = Ae^{-ax^2},$$

then

$$\begin{aligned}1 &= \int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = A^2 \int_{-\infty}^{\infty} e^{-2ax^2} \, dx, \\ A^2 \sqrt{\frac{\pi}{2a}} &= 1, \\ A &= \left(\frac{2a}{\pi}\right)^{1/4}.\end{aligned}$$

1.4 Trigonometric identities

Concept

$$\begin{aligned}\sin^2 \theta &= \frac{1}{2}(1 - \cos 2\theta), & \cos^2 \theta &= \frac{1}{2}(1 + \cos 2\theta), \\ \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B, \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B, \\ \tan(2\theta) &= \frac{2 \tan \theta}{1 - \tan^2 \theta}.\end{aligned}$$

8 Black-Body Radiation and Photoelectric Effect

8.1 Stefan-Boltzmann law and Wien law

Concept

Stefan-Boltzmann law:

$$P = \sigma AT^4.$$

For a spherical star:

$$A = 4\pi R^2.$$

Therefore:

$$T = \left(\frac{P}{\sigma 4\pi R^2} \right)^{1/4}.$$

Wien displacement law:

$$\lambda_{\max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}.$$

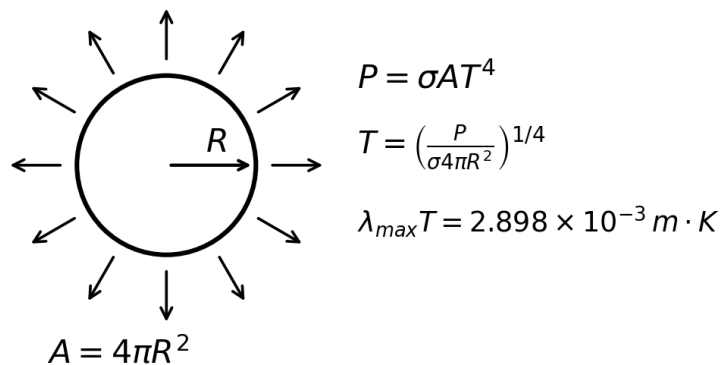


Figure 8.1: Clean star luminosity diagram for black-body radiation.

Example: blue giant star

Given

$$P = 4.5 \times 10^{30} \text{ W}, \quad R = 8.2 \times 10^9 \text{ m}, \quad \sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4.$$

Temperature

$$T = \left[\frac{4.5 \times 10^{30}}{(5.67 \times 10^{-8})(4\pi)(8.2 \times 10^9)^2} \right]^{1/4}.$$

$$T = 1.75 \times 10^4 \text{ K}$$

Peak wavelength

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}}{1.75 \times 10^4}.$$

$$\lambda_{\max} = 1.66 \times 10^{-7} \text{ m} = 166 \text{ nm}$$

8.2 Photoelectric effect

Concept

Photon energy:

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}.$$

Using $hc = 1240 \text{ eV nm}$:

$$E_{\text{photon}} = \frac{1240}{\lambda(\text{nm})} \text{ eV}.$$

Einstein photoelectric equation:

$$K_{\max} = E_{\text{photon}} - \phi.$$

Example: two metals illuminated by the same light

Metal A:

$$\phi_A = 4.2 \text{ eV}, \quad K_A = 1.2 \text{ eV}.$$

Photon energy:

$$\begin{aligned} E_{\text{photon}} &= K_A + \phi_A \\ &= 1.2 + 4.2 = 5.4 \text{ eV}. \end{aligned}$$

Wavelength:

$$\lambda = \frac{1240 \text{ eV nm}}{5.4 \text{ eV}}.$$

$$\lambda = 230 \text{ nm}$$

For Metal B:

$$K_B = 2.8 \text{ eV}.$$

Work function:

$$\begin{aligned} \phi_B &= E_{\text{photon}} - K_B \\ &= 5.4 - 2.8. \end{aligned}$$

$$\phi_B = 2.6 \text{ eV}$$

9 Compton Scattering

9.1 Compton wavelength shift

Concept

Compton shift:

$$\lambda' = \lambda + \frac{h}{m_e c} (1 - \cos \theta).$$

where

$$\frac{h}{m_e c} = 0.00243 \text{ nm}.$$

Photon energy:

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{\lambda(\text{nm})}.$$

Kinetic energy transferred to electron:

$$K_e = E_i - E'.$$

Example: Compton scattering calculation

Scattered wavelength

$$\lambda' = 0.2700 + 0.00243(1 - \cos 65^\circ) \text{ nm}.$$

$$\lambda' = 0.271 \text{ nm}$$

Scattered photon energy

$$E' = \frac{1240}{0.2714} \text{ eV}.$$

$$E' = 4.57 \text{ keV}$$

Kinetic energy of electron

$$E_i = \frac{1240}{0.2700} \text{ eV} = 4.59 \text{ keV},$$
$$K_e = E_i - E' = 4.593 - 4.569 \text{ keV}.$$

$$K_e \approx 24 \text{ eV}$$

10 Quantum Mechanics: Particle in a Box

10.1 Infinite square well

Concept

For

$$0 \leq x \leq L, \quad V(x) = 0,$$

Schrodinger equation becomes

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi.$$

Rearrange:

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0, \quad k^2 = \frac{2mE}{\hbar^2}.$$

General solution:

$$\psi(x) = A \sin(kx) + B \cos(kx).$$

Boundary conditions:

$$\psi(0) = 0 \Rightarrow B = 0, \quad \psi(L) = 0 \Rightarrow \sin(kL) = 0.$$

Therefore:

$$kL = n\pi, \quad k = \frac{n\pi}{L}.$$

Energy:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2}.$$

Normalized wavefunction:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right).$$

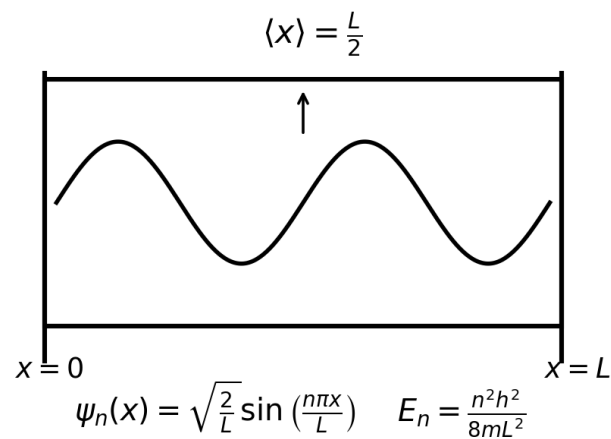


Figure 10.1: Clean particle-in-a-box sketch used for normalization and expectation values.

10.2 Symmetric infinite well about the origin

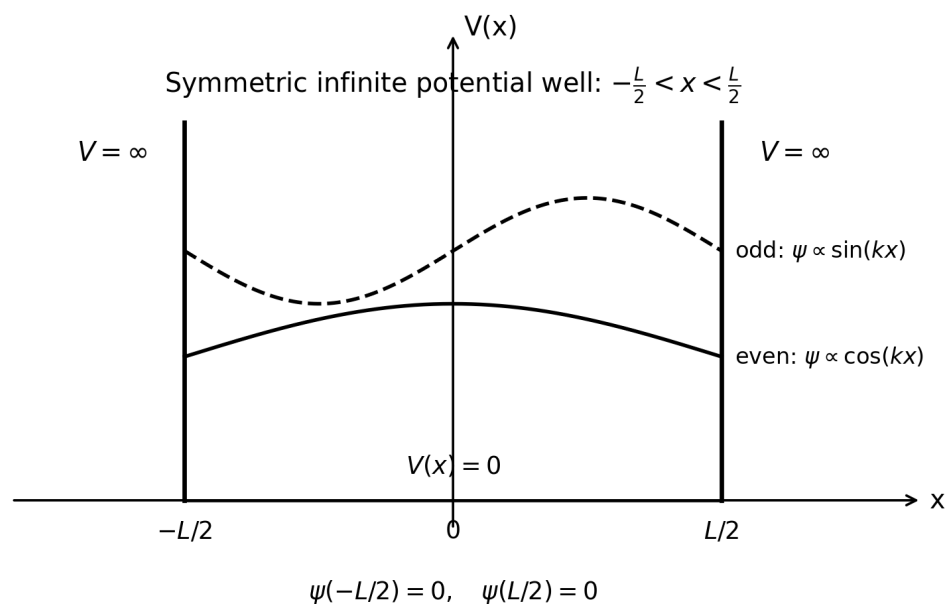


Figure 10.2: Clean symmetric infinite-well diagram showing the boundaries $-L/2$ and $L/2$, with even and odd states.

Example: symmetric infinite potential well

An electron is confined in the one-dimensional infinite potential well

$$V(x) = \begin{cases} 0, & -\frac{L}{2} < x < \frac{L}{2}, \\ \infty, & \text{otherwise.} \end{cases}$$

Find the normalized wavefunctions for both odd and even states and derive the quantized energy levels.

Inside the well

$$V(x) = 0.$$

The time-independent Schrodinger equation becomes

$$-\frac{\hbar^2}{2m_e} \frac{d^2\psi}{dx^2} = E\psi.$$

Rearrange:

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0, \quad k^2 = \frac{2m_e E}{\hbar^2}.$$

General solution:

$$\psi(x) = A \sin(kx) + B \cos(kx).$$

Boundary conditions:

$$\psi\left(-\frac{L}{2}\right) = 0, \quad \psi\left(\frac{L}{2}\right) = 0.$$

Odd states

For odd states:

$$\psi_{\text{odd}}(x) = A \sin(kx).$$

Apply the boundary condition at $x = L/2$:

$$\psi_{\text{odd}}\left(\frac{L}{2}\right) = 0.$$

$$A \sin\left(\frac{kL}{2}\right) = 0.$$

For $A \neq 0$:

$$\sin\left(\frac{kL}{2}\right) = 0.$$

Therefore

$$\frac{kL}{2} = n\pi, \quad k = \frac{2n\pi}{L}, \quad n = 1, 2, 3, \dots$$

So

$$\psi_{\text{odd},n}(x) = A \sin\left(\frac{2n\pi x}{L}\right).$$

Normalize:

$$\int_{-L/2}^{L/2} |\psi_{\text{odd},n}(x)|^2 dx = 1.$$

$$A^2 \int_{-L/2}^{L/2} \sin^2\left(\frac{2n\pi x}{L}\right) dx = 1.$$

Using

$$\int_{-L/2}^{L/2} \sin^2\left(\frac{2n\pi x}{L}\right) dx = \frac{L}{2},$$

we get

$$A^2 \frac{L}{2} = 1, \quad A = \sqrt{\frac{2}{L}}.$$

$$\psi_{\text{odd},n}(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2n\pi x}{L}\right)$$

Even states

For even states:

$$\psi_{\text{even}}(x) = B \cos(kx).$$

Apply the boundary condition at $x = L/2$:

$$\psi_{\text{even}}\left(\frac{L}{2}\right) = 0.$$

$$B \cos\left(\frac{kL}{2}\right) = 0.$$

For $B \neq 0$:

$$\cos\left(\frac{kL}{2}\right) = 0.$$

Therefore

$$\frac{kL}{2} = \frac{(2n-1)\pi}{2}, \quad k = \frac{(2n-1)\pi}{L}, \quad n = 1, 2, 3, \dots$$

So

$$\psi_{\text{even},n}(x) = B \cos\left(\frac{(2n-1)\pi x}{L}\right).$$

Normalize:

$$\int_{-L/2}^{L/2} |\psi_{\text{even},n}(x)|^2 dx = 1.$$

$$B^2 \int_{-L/2}^{L/2} \cos^2\left(\frac{(2n-1)\pi x}{L}\right) dx = 1.$$

Using

$$\int_{-L/2}^{L/2} \cos^2\left(\frac{(2n-1)\pi x}{L}\right) dx = \frac{L}{2},$$

we get

$$B^2 \frac{L}{2} = 1, \quad B = \sqrt{\frac{2}{L}}.$$

$$\psi_{\text{even},n}(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{(2n-1)\pi x}{L}\right)$$

Energy levels

From

$$k^2 = \frac{2m_e E}{\hbar^2},$$

we get

$$E = \frac{\hbar^2 k^2}{2m_e}.$$

For the combined sequence,

$$k_n = \frac{n\pi}{L}, \quad n = 1, 2, 3, \dots$$

Therefore

$$E_n = \frac{\hbar^2}{2m_e} \left(\frac{n\pi}{L}\right)^2.$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2m_e L^2} = \frac{n^2 \hbar^2}{8m_e L^2}$$

Combined final form

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right), & n = 1, 3, 5, \dots, \\ \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), & n = 2, 4, 6, \dots \end{cases}$$

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right), & n = 1, 3, 5, \dots, \\ \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), & n = 2, 4, 6, \dots, \end{cases} \quad E_n = \frac{n^2 \pi^2 \hbar^2}{2m_e L^2}$$

10.3 Normalization of ψ_n **Example: normalize the wavefunction**

$$\int_0^L |\psi_n(x)|^2 dx = 1.$$

Substitute:

$$A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1.$$

Use

$$\sin^2 u = \frac{1}{2}(1 - \cos 2u).$$

Then

$$A^2 \int_0^L \frac{1}{2} \left[1 - \cos\left(\frac{2n\pi x}{L}\right) \right] dx = 1.$$

The cosine integral is zero over 0 to L , so

$$A^2 \frac{L}{2} = 1.$$

$$A = \sqrt{\frac{2}{L}}$$

10.4 Expectation value of position**Example: first excited state**

$$\psi_2(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right).$$

Expectation value:

$$\langle x \rangle = \int_0^L x |\psi_2(x)|^2 dx$$

$$= \frac{2}{L} \int_0^L x \sin^2 \left(\frac{2\pi x}{L} \right) dx.$$

Use

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A).$$

Then

$$\begin{aligned} \langle x \rangle &= \frac{1}{L} \int_0^L x \left[1 - \cos \left(\frac{4\pi x}{L} \right) \right] dx \\ &= \frac{1}{L} \left[\int_0^L x dx - \int_0^L x \cos \left(\frac{4\pi x}{L} \right) dx \right]. \end{aligned}$$

Let

$$k = \frac{4\pi}{L}.$$

Use

$$\int x \cos(kx) dx = \frac{x \sin(kx)}{k} + \frac{\cos(kx)}{k^2}.$$

Over 0 to L :

$$\int_0^L x \cos \left(\frac{4\pi x}{L} \right) dx = 0.$$

Therefore:

$$\langle x \rangle = \frac{1}{L} \left[\frac{x^2}{2} \right]_0^L = \frac{1}{L} \frac{L^2}{2}.$$

$$\boxed{\langle x \rangle = \frac{L}{2}}$$

10.5 Energy transition

Example: energy transition in the box

Formula

$$\Delta E = E_4 - E_1 = \frac{(4^2 - 1^2)h^2}{8mL^2}.$$

Substitution

$$\Delta E = \frac{15(6.626 \times 10^{-34})^2}{8(9.11 \times 10^{-31})(5 \times 10^{-9})^2}.$$

$$\boxed{\Delta E = 3.61 \times 10^{-20} \text{ J}}$$

10.6 Expectation value of momentum

Example: momentum expectation value

Momentum operator:

$$\hat{P}_x = -i\hbar \frac{d}{dx}.$$

For $n = 2$:

$$\psi_2(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right).$$

Expectation value:

$$\langle P_x \rangle = \int_0^L \psi_2^*(x) \left(-i\hbar \frac{d}{dx}\right) \psi_2(x) dx.$$

Since ψ_2 is real:

$$\langle P_x \rangle = -i\hbar \int_0^L \psi_2(x) \frac{d\psi_2(x)}{dx} dx.$$

Compute derivative:

$$\frac{d\psi_2}{dx} = \sqrt{\frac{2}{L}} \frac{2\pi}{L} \cos\left(\frac{2\pi x}{L}\right).$$

Thus

$$\langle P_x \rangle = -i\hbar \left(\frac{2}{L}\right) \left(\frac{2\pi}{L}\right) \int_0^L \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi x}{L}\right) dx.$$

Use

$$\sin A \cos A = \frac{1}{2} \sin(2A).$$

Then

$$\begin{aligned} \int_0^L \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi x}{L}\right) dx &= \frac{1}{2} \int_0^L \sin\left(\frac{4\pi x}{L}\right) dx \\ &= 0. \end{aligned}$$

$$\langle P_x \rangle = 0$$

12.1 Hydrogen ground-state radial probability

Concept

For hydrogen ground state:

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}.$$

Radial probability density:

$$P(r) = 4\pi r^2 |\psi_{100}|^2.$$

Thus

$$P(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0}$$

Example: most probable radius and probability from 0 to a_0

Most probable radius occurs when

$$\frac{dP}{dr} = 0.$$

Since

$$P(r) = \frac{4}{a_0^3} r^2 e^{-2r/a_0},$$

$$\frac{dP}{dr} = \frac{4}{a_0^3} e^{-2r/a_0} \left(2r - \frac{2r^2}{a_0} \right) = 0.$$

$$2r \left(1 - \frac{r}{a_0} \right) = 0.$$

$$r = 0 \quad \text{or} \quad r = a_0.$$

The most probable nonzero radius is

$$r = a_0$$

Probability from 0 to a_0 :

$$P(0 < r < a_0) = \int_0^{a_0} \frac{4r^2}{a_0^3} e^{-2r/a_0} dr.$$

Let

$$x = \frac{2r}{a_0}, \quad r = \frac{a_0 x}{2}, \quad dr = \frac{a_0}{2} dx.$$

Limits:

$$r = 0 \Rightarrow x = 0, \quad r = a_0 \Rightarrow x = 2.$$

$$P = \frac{1}{2} \int_0^2 x^2 e^{-x} dx.$$

Using

$$\int x^2 e^{-x} dx = -e^{-x}(x^2 + 2x + 2),$$

$$P = \frac{1}{2} \left[-e^{-x}(x^2 + 2x + 2) \right]_0^2.$$

$$P = \frac{1}{2} \left[-10e^{-2} + 2 \right] = 1 - 5e^{-2}.$$

$$P = 0.323$$

Orbital Angular Momentum - Test 2 Support

Concept

Total orbital angular momentum:

$$L = \sqrt{\ell(\ell + 1)} \hbar$$

Allowed z-components:

$$L_z = m_\ell \hbar, \quad m_\ell = -\ell, \dots, 0, \dots, +\ell$$

Worked example: $\ell = 2$

Step 1: Substitute $\ell = 2$

$$L = \sqrt{2(2 + 1)} \hbar = \sqrt{6} \hbar$$

Step 2: List allowed magnetic quantum numbers

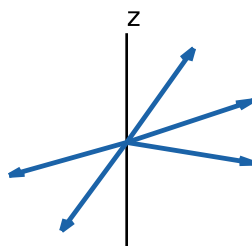
$$m_\ell = -2, -1, 0, 1, 2$$

Step 3: Therefore the allowed z-components are

$$L_z = -2\hbar, -\hbar, 0, \hbar, 2\hbar$$

Degeneracy:

States with different m_ℓ can share the same energy when the energy is independent of m_ℓ .



For $\ell = 2$, the five allowed values are $m_\ell = -2, -1, 0, 1, 2$.

REFERENCES & STUDY SOURCES

Quantum & Optoelectronic Physics 5 - Test 1 + Test 2 Complete Preparation

- 1** CPUT QOE580S tutorial sheets and lecturer memos for propagation of light, polarization, reflection/refraction, optical fibre and laser theory.
- 2** QOE580S past assessment papers and supplied Test 1 / Test 2 materials, including 2023-2024 papers and the 7 October 2026 Test 1.
- 3** Chapter 5 Laser Theory and Chapter 6 Introduction to Quantum Physics lecture material supplied with the course.
- 4** Schrodinger-equation, atomic-physics and quantum-mechanics notes, assignments, memos and worked-problem documents supplied for revision.
- 5** Formula sheets, subject-guide material and selected Physics 5 worked examples used only where they support the confirmed Test 1 or Test 2 scope.
- 6** HNK study-documentation layout adapted into this calculation-focused Test 1 + Test 2 learning edition.

SCOPE NOTE

This combined book keeps Test 1 and Test 2 as separate revision sections. Test 1 focuses on light propagation, polarization, reflection, refraction and optical fibre. Test 2 focuses on laser theory, quantum physics and Schrodinger applications. Duplicate questions are consolidated while complete calculations, figures and intermediate steps are retained.

HNK LEARNING, PROJECT & CONTACT RESOURCES

MindBuilders | South Africa | HNK IoT Solutions

Learning & project websites

Primary website

<https://hnkiot.com/>

Engineering portfolio

<https://hnk-engineering-portfolio.netlify.app/>

LinkedIn article

<https://lnkd.in/p/dWhpzZPa>

Learn Vectors

<https://learn-vectors.netlify.app/>

Engineering Maths 5

<https://learn-mathseng5.netlify.app/>

AI Learning Hub

<https://learn-ailearninghub.netlify.app/>

UI Builders

<https://uibuilders.netlify.app/>

Canvas / WebGL / Three.js

<https://canvas-webglthreejs.netlify.app/>

HNK IoT Solutions

Website

<https://hnkiot.com/>

Email

<mailto:hello@hnkiot.com>

WhatsApp

<https://wa.me/27614719400>

GitHub

<https://github.com/hnkiot>

LinkedIn

<https://www.linkedin.com/in/hnk-iot-solutions/?isSelfProfile=false>

Personal / creator profiles

GitHub

<https://github.com/henockhnk092-dot>

YouTube

<https://www.youtube.com/@HNK2005/playlists>

TikTok - Coding Fever

<https://www.tiktok.com/@codingfever>

Instagram

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TikTok - Horizon HNK

<https://www.tiktok.com/@horizonhnk>

X / Twitter

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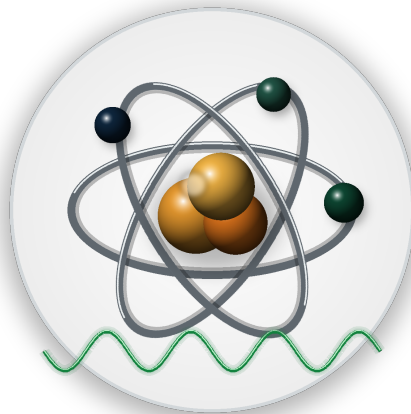
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CONTACT

WhatsApp: +27 61 471 9400 | hello@hnkiot.com | hnkiot.com

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SUMMARY

A complete Test 1 + Test 2 roadmap for Quantum & Optoelectronic Physics 5, combining optics, lasers and quantum mechanics with calculation-first worked solutions.

SUBTITLE

Master the method through complete questions, diagrams, formulas, unit conversions, derivations and full mathematical steps.

AUTHOR INFORMATION

HNK study edition for engineering learners. Structured for revision, printing and later conversion into a responsive online course.

From Maxwell waves to photons, lasers and Schrodinger equations:
learn the method, show every step, and become ready.

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